Marcus Pivato

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Education: Ph.D. in Mathematics, University of Toronto (2001).
B.Sc. in Mathematics, University of Alberta (1994).

Current Positions: Associate Professor, Department of Mathematics, Trent University.
Member, Graduate Program in *Applications of Modeling in the Natural and Social Sciences*, Trent University.
Member, Editorial Board for *Journal of Cellular Automata.*

Research Interests: Social choice theory, decision theory and game theory.

Awards and Honours

2008-2013: NSERC Discovery Grant ($90,000).
2006: Promoted to Associate Professor at Trent University.
2005: Merit Award for research and university service (Trent U.)
2004: Nominated for Symons Teaching Award (Trent U.)
2003-2008: NSERC Discovery Grant ($50,000).
At the University of Toronto:
2000-2001: MITACS Research Assistantship.
1999-2000: Ontario Graduate Scholarship.
1995-1999: NSERC Postgraduate Scholarship.
At the University of Alberta:
1994: Dean's Silver Medal in Science.
1990-1994: Canada Scholarship.
1993-1994: Murray Thomas Gibson Award in Mathematics.
1992: NSERC Undergraduate Award.

Textbook. *Linear Partial Differential Equations and Fourier Theory*, 630 pages, Cambridge University Press, 2010. ISBN: 978-0521136594

Publications in social choice and decision theory

[1] *Voting rules as statistical estimators*, M. Pivato, *Social Choice & Welfare* **40** (#2), February 2013, , pp. 581-630.

[2] *Risky social choice with incomplete or noisy interpersonal comparisons of well-being,* M. Pivato, *Social Choice & Welfare* **40** (#1), January 2013, pp 123-139.

[3] *A statistical approach to epistemic democracy*, M. Pivato; *Episteme* **9**, special issue #2, June 2012, pp 115-137.

[**4**] *Incoherent majorities: The McGarvey problem in judgement aggregation*, Klaus Nehring and M. Pivato, *Discrete Applied Mathematics* **159** (2011), pp.1488-1507.

[**5**] *Geometric models of consistent judgement aggregation*, M. Pivato, *Social Choice & Welfare* **33** (#4), 2009, pp.559-574.

[**6**] *Pyramidal Democracy*, M. Pivato, *Journal of Public Deliberation*, Vol. **5** (#1), 2009, Article 8.

[**7**] *Twofold optimality of the relative utilitarian bargaining solution,* M. Pivato, *Social Choice & Welfare* **32** (#1), 2009, pp.79-92.

Revise and Resubmit

(8) *Condorcet admissibility: Indeterminacy and path-dependence under majority voting on interconnected decisions*, by Klaus Nehring, M. Pivato, and Clemens Puppe; originally submitted to *Theoretical Economics* in August, 2011 (60 pages).

(9) *Variable population voting rules*, M. Pivato; resubmitted to *Journal of Mathematical Economics*, November 2012 (26 pages).

Submitted

(10) *Multitutility representations for incomplete difference preorders* (23 pages), submitted to *Theory and Decision*, December 2012.

(11) *Formal utilitarianism, range voting, and approval voting*, submitted to *Social choice & Welfare*, November 2012 (15 pages).

(12) *Social choice with approximate interpersonal comparisons of welfare gains I: utilitarian models*, M. Pivato; submitted to *Theory & Decision*, September 2012 (30 pages).

(13) *Social choice with approximate interpersonal comparisons of welfare gains II: non-utilitarian models*, M. Pivato; submitted to *Theory & Decision*, September 2012 (30 pages).

(14) *Social welfare with incomplete ordinal interpersonal comparisons*, M. Pivato; submitted to *Journal of Mathematical Economics*, June 2012 (18 pages).

(15) *Additive representation of separable preferences on infinite products*, M. Pivato; submitted to *Journal of Mathematical Economics*, December, 2011 (27 pages).

(16) *A fair pivotal mechanism for nonpecuniary public goods,* by M. Pivato; submitted to *Journal of Public Economic Theory*, May, 2012 (23 pages).

In preparation

(17) *Majority rule in the absence of a majority,* by Klaus Nehring and M. Pivato (50 pages).

(18) *Additive majority rules in judgement aggregation*, by Klaus Nehring and M. Pivato (40 pages).

(19) *Ranking Multidimensional Alternatives and Uncertain Prospects*, by Philippe Mongin and M. Pivato

(20) *Robust coexistence of formal and informal markets*, by Nejat Anbarci, Pedro Gomis-Porqueras, and M. Pivato.

Publications in dynamical systems and probability theory

⟨**1**⟩ *The ergodic theory of cellular automata*, M. Pivato, *International Journal of General Systems*, **41** #6 (2012)

⟨**2**⟩ *Positive expansiveness versus network dimension in symbolic dynamical systems*, M. Pivato, *Theoretical Computer Science* **412** #30 (2011), pp.3838-3855.

⟨**3**⟩ *RealLife*, by M. Pivato. In: Andrew Adamatzky (Ed.), *Game of Life Cellular Automata* (Springer-Verlag, 2010), Chapter 12, pp.223-234.

⟨**4**⟩ *Emulating Bratteli-Vershik adic systems using cellular automata*, M. Pivato and Reem Yassawi. *Ergodic Theory & Dynamical Systems*, (2010), **30**, pp.1561-1572.

⟨**5**⟩ *Emergent defect dynamics in two-dimensional cellular automata*, by Martin Delacourt and M. Pivato, *Journal of Cellular Automata*, **4** (#2), 2009, pp.111-124.

⟨**6**⟩ *The ergodic theory of cellular automata*, in the *Encyclopedia of Complexity and System Science*, Robert A. Meyers, ed. (Springer-Verlag, 2009).
[`http://www.springer.com/physics/complexity/book/978-0-387-75888-6`]

⟨**7**⟩ *Module shifts and measure rigidity in linear cellular automata,* M. Pivato, *Ergodic Theory & Dynamical Systems*, **28**, (#6), December 2008, pp.1945-1958.

⟨**8**⟩ *The spatial structure of odometers in certain cellular automata.* by M. Pivato and Reem Yassawi. Proceedings of *Journées Automates Cellulaires* (Uzès, France; April 21-25, 2008), pp.119-129.

⟨**9**⟩ *Defect particle kinematics in one-dimensional cellular automata*, M. Pivato, *Theoretical Computer Science*, **377**, (#1-3), May 2007, pp.205-228.

⟨**10**⟩ *RealLife: the continuum limit of Larger than Life cellular automata*, M. Pivato, *Theoretical Computer Science*, **372** (#1), 2007, pp. 46-68.

⟨**11**⟩ *Prevalence of odometers in cellular automata*, Ethan M. Coven, M. Pivato, and Reem Yassawi. *Proceedings of the American Mathematical Society*, **135**, 2007, pp.815-821.

⟨**12**⟩ *Spectral domain boundaries in cellular automata*, M. Pivato, *Fundamenta Informaticae*, **78** (#3), 2007, pp.417-447.

⟨**13**⟩ *Algebraic invariants for crystallographic defects in cellular automata*, M. Pivato, *Ergodic Theory & Dynamical Systems*, **27** (#1), February 2007, pp. 199-240.

⟨**14**⟩ *Asymptotic randomization of subgroup shifts by linear cellular automata,* Alejandro Maass, Servet Martinez, M. Pivato, and Reem Yassawi, *Ergodic Theory & Dynamical Systems*, **26** (#4), 2006, pp.1203-1224.

⟨**15**⟩ *Asymptotic randomization of sofic shifts by linear cellular automata,* M. Pivato and Reem Yassawi, *Ergodic Theory & Dynamical Systems* **26** (#4), 2006, pp.1177-1201.

⟨**16**⟩ *Attractiveness of the Haar measure for the action of linear cellular automata in Abelian topological Markov chains*, by A. Maass, S. Martinez, M. Pivato and R. Yassawi, pp. 100-108, in *Dynamics & Stochastics: Festschrift in honour of Michael Keane* (volume 48, Lecture Notes Monograph Series of Institute for Mathematical Statistics), 2006.

⟨**17**⟩ *Cellular automata vs. quasisturmian shifts*, M. Pivato, *Ergodic Theory & Dynamical Systems*, **25** (#5), 2005, pp. 1583-1632.

⟨**18**⟩ *Invariant measures for bipermutative cellular automata,* M. Pivato, *Discrete & Continuous Dynamical Systems A*, **12** (#4), 2005, pp. 723 - 736.

⟨**19**⟩ *Limit measures for affine cellular automata II,* M. Pivato and Reem Yassawi, *Ergodic Theory & Dynamical Systems*, **24** (#6), 2004, pp. 1961-1980

⟨**20**⟩ *Interior symmetry and local bifurcation in coupled cell networks,* by Martin Golubitsky, M. Pivato, and Ian Stewart, *Dynamical Systems: an International Journal*, **19** (#4), 2004, pp.389-407.

⟨**21**⟩ *Symmetry groupoids and patterns of synchrony in coupled cell networks,* by Martin Golubitsky, M. Pivato, and Ian Stewart, *SIAM Journal of Applied Dynamical Systems*, **2** (#4), 2003, pp. 609 - 646.

⟨**22**⟩ *Estimating the spectral measure of a multivariate stable distribution via spherical harmonic analysis*, M. Pivato and L. Seco, *Journal of Multivariate Analysis*, **87** (#2), 2003, pp. 219-240.

⟨**23**⟩ *Multiplicative cellular automata on nilpotent groups: Structure, entropy, and asymptotics,* M. Pivato, *Journal of Statistical Physics*, **110**(#1/2), 2003, pp. 247-267.

⟨**24**⟩ *Conservation laws for cellular automata,* M. Pivato, *Nonlinearity*, **15**, 2002, pp. 1781-1793.

⟨**25**⟩ *Limit measures for affine cellular automata,* M. Pivato and Reem Yassawi, *Ergodic Theory & Dynamical Systems*, **22**(#6), 2002, pp. 1269-1287.

⟨**26**⟩ *Building a stationary stochastic process from a finite-dimensional marginal,* M. Pivato, *Canadian Journal of Mathematics*, **53**(#2), 2001, pp. 382-413.

⟨**27**⟩ *Measures of dependence for multivariate Lévy distributions*, Jeff Boland, Tom Hurd, M. Pivato and Luis Seco, *Disordered and Complex Systems* (London, 2000) pp. 289-295; American Institute of Physics Conference Proceedings vol. **553**.

Doctoral Thesis: *Analytical Methods for Multivariate Stable Probability Distributions,* Department of Mathematics, University of Toronto, 2001. (Supervisor: Luis Seco.)

Invited Talks:

1. *Ranking Multidimensional Alternatives and Uncertain Prospects* (joint work with Philippe Mongin). To be presented at:
   - Special session on "Risk and equity" at the meeting of the Society for the Advancement of Economic Theory, Paris, July 22-27, 2013.
   - Colloquium, Department of Mathematics, London School of Economics, February 21, 2013)
2. *The median rule in judgement aggregation* (joint with K. Nehring), to be presented at the *AMS Special Session on the Mathematics of Decisions, Elections, and Games* at the 2013 Joint Mathematics Meetings, San Diego, January, 2013.
3. *A fair pivotal mechanism for nonpecuniary public goods.* Presented at:
   - 13th meeting of the Association for Public Economic Theory (PET 12), Taipei, June 12-15, 2012. [`http://[www.accessecon.com/pubs/PET12`].

- *Second World Congress of the Public Choice Society*, in Miami, March 8-11, 2012. [http://http://www.pubchoicesoc.org/2012_papers.php].
- Montreal Natural Resources and Environmental Economics Workshop, Université du Quebec à Montréal, February 3, 2012.

4. *Variable population voting rules*, at the *AMS Special Session on the Mathematics of Decisions, Elections, and Games* at the 2012 Joint Mathematics Meetings, Boston, January 5, 2012.
5. *Supermajoritarian efficient judgement aggregation* (joint work with Klaus Nehring)
   - Université de Montréal Informal Economic Theory Seminar, October 2011.
   - Workshop on New Developments in Judgement Aggregation and Voting Theory Freudenstadt, Germany, September 9-11, 2011. [http://vwl1.ets.kit.edu/Workshop_Judgement_Aggregation_and_Voting.php]
   - 10th International Meeting of the Society for Social Choice and Welfare, Moscow, July 2010 [http://www.hse.ru/conf/sscw2010/judgement_aggregation.html]
6. *Quasiutilitarian social choice with approximate interpersonal comparisons of welfare gains*, presented at:
   - Laurier Centennial Conference (AMMCS-2011). Waterloo, Ontario, July 25-29, 2011. [http://www.ammcs2011.wlu.ca]
   - New Directions in Welfare, OECD, Paris, France, July 6-8, 2011. [http://www.open.ac.uk/socialsciences/welfareconomicstheory/]
7. *Additive Representation of Separable Preferences over Infinite Products*, presented at:
   - Université de Montréal Informal Economic Theory Seminar, November 2011.
   - Laurier Centennial Conference (AMMCS-2011). Waterloo, Ontario, July 25-29, 2011. [http://www.ammcs2011.wlu.ca]
   - Meeting of the Society for Economic Design, University of Montreal, June 15-17, 2011. [http://www.cireq.umontreal.ca/activites/110615/sed2011.html]
   - North American Summer Annual Meeting of the Econometric Society, St.Louis, MO, June 9-12, 2011. [http://artsci.wustl.edu/ econconf/EconometricSociety]
   - Department of Mathematics, SUNY Pottsdam, NY, June 3, 2011.
   - Montreal Natural Resources and Environmental Economics Workshop, McGill University, January 14, 2011 [http://www.cireq.umontreal.ca/resenv/resenv11.html]
8. *A statistical approach to epistemic democracy*, Episteme conference, "Social Epistemology meets Formal Epistemology", Carnegie Mellon University, June 24-26, 2011. [http://www.hss.cmu.edu/philosophy/centerformalepistemology/episteme-schedule-2011.php]
9. *The McGarvey Problem in Judgement Aggregation*, Seminar of Laboratoire de combinatoire et d'informatique mathématique (LaCIM) at Université du Quebec à Montréal. April 15, 2011. [http://lacim.uqam.ca/seminaire]

10. *Social choice with approximate interpersonal comparisons of utility*, presented at:
    - Choice Group Seminar of the London School of Economics, February 15, 2011
    - Université de Montréal Economic Theory Reading Group, October 14, 2010
    - 10th International Meeting of the Society for Social Choice and Welfare, Moscow, July 2010. [http://www.hse.ru/conf/sscw2010/judgement_aggregation.html]
11. *Module shifts and measure rigidity in linear cellular automata*, Special Session on *Algebraic Dynamics* at the Joint Meetings of AMS/MAA, San Diego (January, 2008).
12. *Automata 2007*, Fields Institute, Toronto, Canada, August 27-29, 2007.
13. *CanaDAM 2007*, BIRS, Alberta, Canada. May 28-31, 2007.
14. *Defect kinematics in cellular automata* and *RealLife Euclidean automata* (one hour each). Workshop on *Information propagation in cellular automata*, Ecole Normale Supérieure de Lyon, February 19-21, 2007.
15. *Cohomological crystallographic defects in cellular automata*, Southern Ontario Dynamics Day, Fields Institute, Toronto. April 7, 2006.
16. *Spectral crystallographic defects in cellular automata.* Session on *Ergodic Theory* at the Winter 2005 meeting of the Canadian Mathematical Society, Victoria, British Columbia, December 10-12, 2005.
17. *Defect particle dynamics in cellular automata*, Seminar on *Modelling and Computational Science* at the University of Ontario Institute of Technology, Oshawa, November 18, 2005.
18. *Propagating structures in cellular automata.* Two one-hour lectures at Departamento de Ingeniería Matemática, Universidad de Chile, Santiago, October 25-26, 2005.
19. *Crystallographic defects in cellular automata.* Special session on *Measurable, Symbolic, and Tiling Dynamical Systems* of the Eastern Section Meeting of the American Mathematical Society, Bard College, Annandale-on-Hudson, New York, October 8-9, 2005
20. *RealLife: the continuum limit of Larger than Life cellular automata.* Colloquium lecture at the Department of Mathematics & Statistics, University of Guelph, October 7th, 2004
21. *Cellular automata vs. quasisturmian systems*, at the *Fifth international conference on Dynamical Systems and Differential Equations*, California State Polytechnic University, Pomona, June 16 - 19, 2004.
22. *Asymptotic randomization of sofic shifts by linear cellular automata,* at the Joint International Meeting of the American Mathematical Society and the Real Sociedad Matematica Espanola, Seville, Spain, June 18-21, 2003.
23. *Limit measures for affine cellular automata,* at the *Workshop on Dynamics and Randomness*, Departamento de Ingeniería Matemática, Universidad de Chile, Santiago, December, 2000.

Other scholarly activities

Miscellaneous

February 1-28, 2011: Visiting researcher at the Centre for Philosophy of Natural and Social Science at the London School of Economics.

Since 2009: Member of the editorial board for the *Journal of Cellular Automata.*

Since 2008: Member of Working Group 1.5 (*Cellular Automata and Discrete Complex Systems*) of the IFIP (*International Federation for Information Processing*).

In 2006: Invited to write article [**6**] for *Encyclopaedia of Complexity & System Science.*

Research Grant Refereeing

2011: External referee for ECOS-CONICYT (France-Chile)
2009: External referee for NSERC Discovery Grant application #23****
2008: External referee for FONDECYT grant (Chile).
2008: External referee for FONDECYT grant (Chile).
2007: External referee for FONDECYT grant (Chile).
2004: External referee for NSERC Discovery Grant application #23****
2003: External referee for NSERC Discovery Grant application #24***

Conference Organization

2013: Member of Program Committee for *13th Meeting of the Association for Public Economic Theory* (PET13) in Lisbon, Portugal (July 5-7, 2013).

Co-organizer (with Marc Kilgour) of two special sessions at AMMCS-13 conference in Waterloo, Canada (August 26-30, 2013): *Social Choice* and *Games and Decisions.*

2011: Member of Program Committee for *17th International Workshop on Cellular Automata and Discrete Complex Systems* (AUTOMATA 2011).

2004: Organized special session 'Cellular Automata and Multidimensional Symbolic Dynamics' at the *Fifth international conference on Dynamical Systems and Differential Equations* (California State Polytechnic University, Pomona, June 16 - 19, 2004)

Supervision and training of students

2012: External committee member for Ph.D. thesis of Arthur Paul Pedersen, Department of Philosophy at Carnegie Mellon University. (Supervisor: Teddy Seidenfeld.)

August 26, 2011: External examiner on M.Sc. thesis defense of Andrew Kabbes, Department of Mathematics, Wilfrid Laurier University. (Supervisor: Marc Kilgour. Thesis title: *A procedure for fair division of indivisible, identical items with entitlements.*)

2011-2012: Member of M.Sc. supervisory committee for Sarah Bale, AMINSS, Trent University. (Supervisor: Raul Ponce-Hernandez. Thesis title: *Modelling Agroforestry Alternatives to Slash-and-Burn "Milpa" Agriculture in the Yucatán, Mexico, with WaNuLCAS to generate multiple ecosystem services and enhance rural livelihoods in a changing climate.*)

May 1, 2009 to August 31, 2009: Supervised a NSERC USRA recipient, Erik Cameron, in a project entitled, *Susceptibility to strategic voting in median and mean voting systems.*

May 1, 2008 to August 31, 2008: Supervised a NSERC USRA recipient, Erik Cameron, in a project entitled, *Nomos dynamics and self-selective voting rules.*

May 1, 2007 to August 31, 2007: Supervised a visiting graduate student from ENS Lyon, Martin Delacourt, in a project entitled, *Emergent Defect Dynamics in 2-dimensional Cellular Automata.*

September 12, 2006: Chair of M.Sc. thesis defense for Michael Jack in the AMINSS graduate program at Trent University. (Supervisor: Brian Patrick. Title: *Workload Modeling and Internal Backfilling for Parallel Job Scheduling*).

May 1, 2006 to August 31, 2006: Supervised a Herzberg Award recipient, Joshua Grant, in a project entitled, *Dynamics of Voting Networks.*

September 1, 2005 to August 31, 2006: (with Reem Yassawi) co-supervised a postdoctoral fellow, Pierre Tisseur, who did research on cellular automata and automaton networks.

October 28, 2005: External examiner for the Ph.D. defense of Marcelo Sobottka, at the Departamento de Ingeniería Matemática, Universidad de Chile, Santiago. (Supervisors: Alejandro Maass and Servet Martínez. Title: *Representación y Aleatorización en sistemas dinámicos de tipo algebraico*).

May 1 to August 31 2004: Supervised an NSERC Undergraduate Student Research Award (USRA) recipient, Matthew Drescher, in a project entitled, *Emergent Statistical Physics of Particle-Preserving Cellular Automata.*

Journal Refereeing

2012	*Economics Bulletin.*
	Mathematical Social Science.
	Mind.
	Journal of Economic Theory.
	Social Choice & Welfare (4 papers).
	Episteme.
2011	*Social Choice & Welfare* (3 papers).
	Economics and Philosophy.
	Journal of Theoretical Politics.
2010	*Journal of Political Economy.*
	Economics and Philosophy.
2009	*Social Choice & Welfare* (2 papers).
2007	*Theoretical Informatics and Applications.*
	Dynamical Systems.
	Journal of Cellular Automata.
2006	*Nonlinearity.*
	Stochastics and Dynamics.
2005	*Discrete & Continuous Dynamical Systems* A.
	Ergodic Theory & Dynamical Systems (2 papers).
	Topology and its Applications.
2004	*Discrete & Continuous Dynamical Systems* A.
	Journal of Physics A *(Mathematical and General)*
2003	*Discrete & Continuous Dynamical Systems* A.
	Nonlinearity.
	Proceedings of the London Math. Society.

Math Reviews/Zentralblatt

(Zbl 1204.91051) Alcantud, José and Mehta, Ghanshyam B. Constructive utility functions on Banach spaces *J. Math. Anal. Appl.*, 350, No. 2, 590–600 (2009).

(Zbl 1188.37010) Buescu, Jorge. Liapunov stability and the ring of P-adic integers *São Paulo J. Math. Sci.*, 2, No. 1, 77–84 (2008).

(Zbl 1151.91412) Addario-Berry, L.; Reed, B.A. Ballot theorems, old and new. *Horizons of combinatorics.* Bolyai Society Mathematical Studies 17, 9–35 (2008).

(Zbl 1151.91036) Balinski, Michel. Fair majority voting (or how to eliminate gerrymandering). *American Mathematical Monthly*, 115, No. 2, 97-113 (2008).

(Zbl 1169.91008) Tanino, Tetsuzo; Moritani, Atsushi; Tatsumi, Keiji. Coalition formation in convex TU-games based on population monotonicity of random order values. *J. Nonlinear Convex Anal.* 9, No. 2, 273-281 (2008).

(Zbl 1147.37009) Boyle, Mike; Lee, Bryant. Jointly periodic points in cellular automata: Computer explorations and conjectures. *Experimental Mathematics* 16 (2007), no.3, 293-302.

(MR2334506) Biely, Christoly; Dragosits, Klaus; Thurner, Stefan. The prisoner's dilemma on co-evolving networks under perfect rationality. *Phys. D* 228 (2007), no. 1, 40–48.

(MR2304535) Aragonés, Enriqueta. Government formation in a two dimensional policy space. *Internat. J. Game Theory* 35 (2007), no. 2, 151–184.

(MR2285112) Aguiar, Manuela A. D.; Dias, Ana Paula. Minimal coupled cell networks. *Nonlinearity* 20 (2007), no. 1, 193–219.

(MR2260266) Leite, Maria da Conceição A; Golubitsky, Martin. Homogeneous three-cell networks. *Nonlinearity* 19 (2006), no. 10, 2313–2363.

(MR2237146) Elmhirst, Toby; Golubitsky, Martin. Nilpotent Hopf bifurcations in coupled cell systems. *SIAM J. Appl. Dyn. Syst.* 5 (2006), no. 2, 205–251

(MR2151603) Putnam, Ian F. Lifting factor maps to resolving maps. *Israel J. Math.* 146 (2005), 253–280.

Employment

Professor	`Trent University`	07/2002 - present
	(except for research leave, 01/2005 - 06/2005 and 07/2010 - 06/2012)	
Researcher	`Wesleyan University`	01/2005 - 05/2005.
Postdoctoral Fellow	`University of Houston`	08/2001 - 05/2002.
	Studied equivariant dynamics and coupled cell systems.	Supervisor: Martin Golubitsky
Research Assistant	`University of Toronto RiskLab`	07/2000 - 06/2001.
	Investigated risk management methodologies in electricity markets.	Supervisor: Luis Seco

Research interests

My research focuses on collective decision-making and social welfare. In particular, I am interested in judgement aggregation, intergenerational choice and uncertainty, interpersonal utility comparisons, epistemic social choice, bounded rationality and deliberative democracy.

A particularly interesting domain within social choice theory is *judgement aggregation.* Let $\mathcal{P}$ be a set of logically interdependent propositions, and suppose a group of voters must decide the truth/falsehood of each member of $\mathcal{P}$. A *judgement* is an assignment of a truth value (true or false) to each element of $\mathcal{P}$. The set of all judgements can be identified with the Hamming cube $\{\pm 1\}^{\mathcal{P}}$. However, not all judgements are logically consistent, because of the logical interdependencies between elements of $\mathcal{P}$. Only a subset $\mathcal{X} \subset \{\pm 1\}^{\mathcal{P}}$ of judgements are admissible.

Suppose each voter's opinion is a feasible judgement (in $\mathcal{X}$). A *judgement aggregation rule* is a rule for combining the judgements of these voters to produce some feasible collective judgement (also in $\mathcal{X}$). Many social decision problems can be encoded in this framework, including preference aggregation, resource allocation, facility location, committee selection, and object classification. Judgement aggregation problems also arise frequently in the study of machine intelligence. (In this case, the 'voters' are not humans, but autonomous subsystems of some automated decision-making system.)

	A	$A \Rightarrow B$	B
1	T	T	T
2	T	F	F
3	F	T	F
Maj	T	T	F

For example, suppose $\mathcal{P}$ consists of three propositions: A, B, and "$A \Rightarrow B$". Consider three voters $\{1, 2, 3\}$, who must decide the truth or falsehood of these propositions. Suppose the voters have the profile of judgements shown in the table to the right. Each voter has a logically consistent judgement, but the collective judgement generated by proposition-wise majority vote is logically inconsistent (see bottom row of table). This collective inconsistency is not merely an artifact of majority vote. List and Pettit [LP02] have proved an impossibility theorem, which states (roughly) that there is no anonymous, neutral, and 'systematic' procedure which will aggregate any profile of judgements into a logically consistent collective judgement. List and Pettit call this the *Discursive Dilemma.* There is now a burgeoning literature on judgement aggregation; see List and Puppe [LP09] for an excellent survey.

In my paper [**5**] (see page 2 of CV), I introduced the class of 'quasimajoritarian' judgement aggregation rules, which includes propositionwise majority vote, but also includes some rules which use different voting schemes to decide the truth of different propositions. I show that if the profile of voters' beliefs satisfies a condition called 'value restriction', then the output of any quasimajoritarian rule is logically consistent; this generalizes some results of Dietrich and List [DL10]. I then provide two sufficient conditions for value-restriction, defined geometrically in terms of a lattice ordering or a metric structure on the set of voters and propositions. Finally, I introduce another sufficient condition for consistent majoritarian judgement aggregation, called 'convexity'. I show that convexity is not logically related to value-restriction.

In our paper [**4**], Klaus Nehring and I studied the *McGarvey problem* for judgement aggregation: just how many different judgements (including inconsistent ones) can be produced by applying the propositionwise majority voting procedure to given system of propositions? This is question is closely related to the combinatorics and geometry of convex polytopes in very high-dimensional Hamming cubes.

Currently, I am collaborating with Klaus Nehring and Clemens Puppe to search for judgement aggregation rules which best represent the 'majority will' in settings where propositionwise majority vote leads to logical inconsistencies. In our paper **(8)**, we analyse the *Condorcet admissible set*: the set of judgements which agree with the majority in a maximal set of propositions. In the setting of preference aggregation, this corresponds to the *top cycle*. In the setting of *diachronic* judgement aggregation (where propositions are decided sequentially rather than simultaneously, with earlier decisions imposing logical constraints on later decisions), the Condorcet admissible set is the set of all judgements that can be reached through some sequence. We have shown that, for many judgement aggregation problems, this set is quite large —indeed, sometimes it includes *every* logically possible judgement. This means that Condorcet admissibility alone is inadequate as a judgement aggregation principle. It also means that the problem of path-dependency in diachronic judgement aggregation can be very severe.

In our papers **(17)** and **(18)**, Klaus and I study *supermajoritarian efficient* (SME) *judgement aggregation*, a refinement of Condorcet admissibility based on the premise that it is acceptable to overrule a majority on one proposition *only* if this is necessary to agree with a larger supermajority on some other proposition. If $\mathcal{P}$ is the set of logical propositions under consideration, then we can represent the space of logically consistent judgements as a subset $\mathcal{X}$ of the Hamming cube $\{\pm 1\}^{\mathcal{P}}$, which in turn we regard as a subset of the Euclidean space $\mathbb{R}^{\mathcal{P}}$. A very important class of SME rules are the *additive rules*, where the social decision is obtained by solving a linear program on $\mathcal{X}$ defined by the vector of majority margins. This class includes the Slater rule and the Median rule (also called the Kemeny rule). We have axiomatically chararacterized the class of additive rules in terms of supermajoritarian efficiency, separability, upper hemicontinuity and other normative axioms. We have also axiomatically characterized a particular additive rule —the median rule —in terms of a property called *reinforcement*. This can be seen as the judgement aggregation analog of a classic characterization of the Kemeny rule by Young and Levenglick [YL78].

Klaus and I are currently pursuing several further projects which have grown out of this research. This includes: the implications of SME for preference aggregation, the computation of the median rule, and a detailed study of the behaviour and properties of other additive rules.

Intergenerational choice and uncertainty go together for two reasons. First, decisions with very long-term consequences inevitably involve uncertainty, because the future is hard to predict. (A prototypical example is policies concerning climate change.) Second, the formal representation of an intertemporal choice problem (with outcomes indexed by moments in time) is very similar to the formal representation of an uncertainty problem (with outcomes indexed by states of nature), so that a solution method for one problem can often be applied to solve the other problem —or to solve both problems simultaneously. For example, in my paper **(15)**, I use methods from nonstandard analysis and the theory of linearly ordered abelian groups to construct and axiomatically characterize an additive utility representation for any separable, permutation-invariant preference order on an infinite-dimensional Cartesian product of outcome spaces. One interpretation of this model is as infinite-horizon, nondiscounted, intergenerational utilitarian social welfare function. Another interpretation of the same model is as a version of the Savage model of risky decision making, but with a uniformly distributed (hyperreal) probability measure over infinitely many possible states of nature.

I am also investigating how to construct a social welfare order (SWO) when only 'approximate' interpersonal comparisons of well-being are possible. For example, in the paper **(14)**, I suppose that a statement like, 'Alice is happier than Bob' might be meaningful if the psychologies of Alice and Bob are similar enough, or if the difference in their levels of well-being is large enough. This leads to a partial ordering of the set of psychophysical states, which can be used to construct a (partial) social welfare order. From this I obtain an axiomatic characterization of an *approximate maximin* SWO. In the paper **(12)**, I suppose that, instead of approximate comparison of welfare *levels*, we can approximately compare welfare *gains*. For example, in some circumstances, we might be able to say, 'Alice would gain more welfare from eating this bagel than Bob' (e.g. if Alice is starving to death, whereas Bob is well-fed). This is modeled with a partial ordering on the space of personal state transitions, which can be used to construct a partial ordering on the space of social state transitions. From this I obtain an axiomatic characterization of a *quasiutilitarian* ordering.

In the paper [**2**], I consider a model similar to **(14)**, but in the setting of von Neumann-Morgenstern utility functions. In this case, I obtain an axiomatic characterization of an *approximate utilitarian* SWO. The paper also considers a model where the true utility levels of Alice and Bob are hidden variables, about which we have only partial information — I model this by treating their (joint) utility function as a random variable. Using this, I obtain a stochastic, multiprofile version of Harsanyi's Social Aggregation Theorem [Har55]; this provides an argument for the utilitarian SWO in a setting with noisy interpersonal utility comparisons.

In *epistemic* social choice theory, we suppose that there is an objectively correct choice, and each voter receives a 'noisy signal' of the correct choice. The objective of the social planner is to aggregate these signals to make the best possible guess about the correct choice. One way to formalize this idea is to set up a probability model and compute the maximum likelihood estimator (MLE), maximum *a posteriori* estimator (MAP) or expected utility maximizer (EUM), given the data provided by the voters. The prototypical result here is the Condorcet Jury Theorem, which says, under certain plausible conditions, simple majority vote is an MLE for a choice between two alternatives. In the paper [**1**], I show that an abstract voting rule can be interpreted as MLE or MAP if and only if it is a *scoring rule*. (Familiar scoring rules include the plurality rule, the Borda rule, the Kemeny rule, approval voting, and range voting.) I then study distance-based voting rules, in particular, the median rule. Finally, I show that several common 'quasiutilitarian' voting rules (e.g. approval voting) can be interpreted as EUM.

Several of these projects use linearly ordered abelian groups and nonstandard analysis. A linearly ordered abelian group (*loag*) is a set equipped with both an addition operation and a compatible linear ordering. This is the minimum mathematical structure needed to define a cardinal utility function. The set $\mathbb{R}$ of real numbers is a loag, and most classical utility theory considers real-valued utility functions. However, the space $\mathbb{R}^n$ is also a loag (under vector addition and the lexicographical order). $\mathbb{R}^n$-valued cardinal utility functions represent preference relations where some dimensions have 'lexicographical priority' over other dimensions. In the paper **(15)**, I show that many separable preference relations can be represented with a cardinal utility function ranging over a loag. In **(12)** and **(10)**, I use collections of loag-valued cardinal utility functions to represent approximate interpersonal comparisons of welfare gains. In **(9)**, I show that any abstract voting rule which satisfies properties of 'neutrality' and 'reinforcement' can be represented as a scoring rule, where the scoring function takes values in a loag; this extends a result of Myerson [Mye95] by eliminating his Archimedean requirement, and by allowing the space of signals to be infinite. (In the previous paragraph, I gave several examples of $\mathbb{R}$-valued scoring rules; a lexicographical combination of two or more of these rules would yield an $\mathbb{R}^n$-valued scoring rule.)

A particularly interesting loag is the group ${}^*\mathbb{R}$ of *hyperreal* numbers (the subject of nonstandard analysis). One advantage of ${}^*\mathbb{R}$ is that the sum of infinitely many real numbers can be always be represented as an element of ${}^*\mathbb{R}$ in a well-defined way (there is no need to worry about issues of series convergence or integrability). This makes ${}^*\mathbb{R}$ the ideal tool for representing infinite-horizon, nondiscounted utility sums, or for computing expected utility over nonstandard probability spaces. (Nonstandard probability spaces allow events with nonzero-but-infinitesimal probability; this is very useful for representing subgame-perfect equilibria in extensive games with incomplete information, where it is problematic to perform Bayesian conditioning on zero-probability events.) ${}^*\mathbb{R}$-valued functions also arises naturally when we try to define a function through a sequence of real-valued approximations; for example, this argument plays an essential role in the papers **(17)** and **(18)**.

Most social choice models suppose that the voters' opinions are fixed and exogenous; our problem is simply to aggregate these opinions. In contrast, the literature on 'deliberative democracy' argues that voter's beliefs, preferences, and values are formed endogenously through social interaction and information exchange ('deliberation'). By optimally structuring this deliberation process, this literature argues, we can improve the performance of democratic institutions. (Indeed, to the extent that deliberation produces broad consensus on the socially optimal choice, it may render the aggregation stage superfluous.) This is an excellent idea in theory, but in practice it is hard to implement substantive and inclusive deliberation in a polity with millions of people. In the paper [**6**], I propose to make large-scale deliberative democracy feasible through a pyramidal structure of delegation.

It is difficult to construct a mathematical model of a complex collective cognitive activity like deliberation. One problem is that deliberation is inextricably intertwined with issues of incomplete information and bounded rationality. Deliberation is beneficial partly because individual voters are biased, incompletely informed, and incompletely perceive the implications of the information they do have. Thus, an exchange of information and analysis can improve their knowledge and their effective degree of rationality and objectivity. On the other hand, if voters blindly conform to their peer group, or slavishly follow opinion-leaders, then deliberation can lead to irrational 'herding' and 'group-think'. If the electorate splits into disjoint groups, which obtain their information from disjoint ideological 'echo chambers', then deliberation will not lead to broad consensus, but rather, increasingly acrimonious disagreement. Thus, to understand whether a particular deliberative institution will be socially beneficial or not, we must construct a model of boundedly rational opinion formation driven by social interactions. I am currently working on a model of this kind.

Most significant past research contributions

Most of my research from 2000-2009 can be grouped into five areas:

(i) *Emergent defect dynamics in cellular automata.*
(ii) *Asymptotic randomization in algebraic cellular automata.*
(iii) *Measure rigidity in algebraic cellular automata.*
(iv) *Cellular automata vs. highly ordered symbolic systems.*
(v) *Symmetry groupoids in coupled cell networks.*

I will elaborate on each of these areas below.

Background A cellular automaton (CA) is a spatially distributed dynamical system, consisting of an infinite grid of simple, identical, interacting *cells*. Formally, let $D \geq 1$, and let $\mathbb{Z}^D$ be the D-dimensional lattice. Let $\mathcal{A}$ be a finite set, and let $\mathcal{A}^{\mathbb{Z}^D}$ be the space of all $\mathbb{Z}^D$-indexed, $\mathcal{A}$-valued *configurations*. (The space $\mathcal{A}^{\mathbb{Z}^D}$ is compact, metrizable, and zero-dimensional in the Tychonoff topology.) Let $\mathbb{V} \subset \mathbb{Z}^D$ be a finite subset (called a *neighbourhood*); if $\mathbf{a} = [a_{\mathsf{z}}]_{\mathsf{z}\in\mathbb{Z}^D} \in \mathcal{A}^{\mathbb{Z}^D}$, let $\mathbf{a}_{\mathbb{V}} := [a_{\mathsf{v}}]_{\mathsf{v}\in\mathbb{V}} \in \mathcal{A}^{\mathbb{V}}$. If $\mathsf{z} \in \mathbb{Z}^D$, let $\mathbb{V} + \mathsf{z} = \{\mathsf{v} + \mathsf{z}\ ;\ \mathsf{v} \in \mathbb{V}\}$, and treat $\mathbf{a}_{(\mathbb{V}+\mathsf{z})} := [a_{\mathsf{v}+\mathsf{z}}]_{\mathsf{v}\in\mathbb{V}}$ as a member of $\mathcal{A}^{\mathbb{V}}$. If $\phi: \mathcal{A}^{\mathbb{V}} \longrightarrow \mathcal{A}$, then the ($D$-dimensional) *cellular automaton* $\Phi: \mathcal{A}^{\mathbb{Z}^D} \longrightarrow \mathcal{A}^{\mathbb{Z}^D}$ with *local map* ϕ is defined by $\Phi(\mathbf{a})_{\mathsf{z}} := \phi(\mathbf{a}_{\mathbb{V}+\mathsf{z}})$, for all $\mathsf{z} \in \mathbb{Z}^D$ and $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}$.

If $\mathsf{v} \in \mathbb{Z}^D$, then the *shift map* $\boldsymbol{\sigma}^{\mathsf{v}} : \mathcal{A}^{\mathbb{Z}^D} \longrightarrow \mathcal{A}^{\mathbb{Z}^D}$ is defined: $\boldsymbol{\sigma}^{\mathsf{v}}(\mathbf{a}) = [a'_{\mathsf{z}}]_{\mathsf{z}\in\mathbb{Z}^D}$, where $a'_{\mathsf{z}} = a_{\mathsf{z}+\mathsf{v}}$ for all $\mathsf{z} \in \mathbb{Z}^D$. A function $\Phi : \mathcal{A}^{\mathbb{Z}^D} \longrightarrow \mathcal{A}^{\mathbb{Z}^D}$ is a CA if and only if Φ is continuous and commutes with all shifts [Hed69]. A *subshift* is a closed, shift-invariant subset $\mathbf{X} \subset \mathcal{A}^{\mathbb{Z}^D}$. If $\mathbf{X} \subset \mathcal{A}^{\mathbb{Z}^D}$ is a subshift, then for any finite $\mathbb{K} \subset \mathbb{Z}^D$, let $\mathbf{X}_{\mathbb{K}} := \{\mathbf{x}_{\mathbb{K}}\ ;\ \mathbf{x} \in \mathbf{X}\}$ be the set of all $\mathbf{X}$-*admissible* $\mathbb{K}$-*blocks*. If $\mathsf{z} \in \mathbb{Z}^D$, then $\mathbf{X}_{\mathbb{K}} = \mathbf{X}_{\mathbb{K}+\mathsf{z}}$ (because $\mathbf{X}$ is shift-invariant). We say $\mathbf{X}$ is a *subshift of finite type* (SFT) if there is some finite *neighbourhood* $\mathbb{K} \subset \mathbb{Z}^D$ such that $\mathbf{X} = \{\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}\ ;\ \mathbf{a}_{\mathbb{K}+\mathsf{z}} \in \mathbf{X}_{\mathbb{K}},\ \forall \mathsf{z} \in \mathbb{Z}^D\}$. Examples: **(a)** The support of a stationary Markov chain with statespace $\mathcal{A}$ is an SFT in $\mathcal{A}^{\mathbb{Z}}$ (called a *Markov shift*). **(b)** Any spatially periodic pattern (e.g. checkerboard, stripes, etc.) corresponds to an SFT of $\mathcal{A}^{\mathbb{Z}^2}$. **(c)** If the elements of $\mathcal{A}$ are *Wang tiles* (square tiles with edge-matching constraints), then the set of admissible tilings of $\mathbb{Z}^2$ by $\mathcal{A}$ is an SFT of $\mathcal{A}^{\mathbb{Z}^2}$. **(d)** If $\Phi : \mathcal{A}^{\mathbb{Z}^D} \longrightarrow \mathcal{A}^{\mathbb{Z}^D}$ is a CA, then $\mathsf{Fix}[\Phi] := \{\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}\ ;\ \Phi(\mathbf{a}) = \mathbf{a}\}$ is an SFT of $\mathcal{A}^{\mathbb{Z}^D}$.

(i) Emergent Defect Dynamics in CA. In many CA, almost any initial condition rapidly 'coalesces' into large, homogeneous 'domains' —each exhibiting some regular spatial pattern —separated by *defects*. These defects evolve and propagate over time, occasionally colliding and interacting with each other. In one-dimensional CA, such *emergent defect dynamics* (EDD) has been studied both empirically [Gra84, BNR91, CH92, CH93a, CH93b, CH97] and theoretically [Lin84, Elo93a, Elo93b, Elo94, CHS01, KM00, KM02, Kůr03, Kůr05] for more than two decades, but it is still incompletely understood. Even less is known about EDD in multidimensional CA. Indeed, there is not even a consensus about the correct definition of 'domain' vs. 'defect'

In my publications ⟨**5, 9, 12, 13**⟩ (see page 3 of CV), I define 'defects' relative to some reference subshift $\mathbf{X}$. For example, suppose $\mathbf{X}$ is an SFT defined by a neighbourhood $\mathbb{K}$. If $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}$, then the $\mathbf{X}$-*admissible region* of $\mathbf{a}$ is $\mathbb{A} := \{\mathsf{z} \in \mathbb{Z}^D \; ; \; \mathbf{a}_{\mathbb{K}+\mathsf{z}} \in \mathbf{X}_{\mathbb{K}}\}$ (the part covered by the 'regular pattern' encoded in $\mathbf{X}$). Thus, $\mathbb{Z}^D \setminus \mathbb{A}$ is the *defective* part of $\mathbf{a}$. (A similar definition exists if $\mathbf{X}$ is *not* of finite type, but it is more complicated, because of the presence of long-range constraints in the structure of $\mathbf{X}$; see my publications ⟨**12**, §1⟩ or ⟨**13**, §1⟩.)

Defect Kinematics. If $D = 1$ and $\mathbf{X} \subset \mathcal{A}^{\mathbb{Z}}$ is an SFT, then a defect is a finite object, like a 'particle'. Let $\mathbf{L}, \mathbf{R} \subset \mathbf{X}$ be the transitive components of $\mathbf{X}$ to the left and right of the defect. In publication ⟨**9**⟩, I showed that defect particle motion falls into several *kinematic regimes*, depending on $\mathbf{L}$ and $\mathbf{R}$, as shown in the table to the right.

Defect Kinematic Regimes		Right Side (σ,Φ)-Dynamics			
	σ-dynamics	σ-periodic ⇓	Right-regular	Nonzero σ-Entropy	
σ-dynamics	Φ-dynamics	Φ-Periodic	Right-resolving	Φ-Periodic	Anything else
Left Side (σ,Φ)-Dynamics: σ-periodic => Φ-Periodic		Ballistic	Diffusive	Autonomous Pushdn Autom	Complicated
Left-regular	Left-resolving	Diffusive	Diffusive	Markov Pushdn Autom	Complicated
Nonzero σ-Entropy	Φ-Periodic	Autonomous Pushdn Autom	Markov Pushdn Autom	Turing Machine	Complicated
	Anything else	Complicated	Complicated	Complicated	

In the *ballistic* regime, the defect particle moves deterministically with a constant average velocity; this is seen e.g. in elementary CA #54, #62, #110, and #184. In the *diffusive* regime, the particle executes a random walk; this is closely related to the work of [Elo93a, Elo93b, Elo94]. In the *pushdown automata* regimes, the particle can be described as a simple computer with a pushdown 'stack' memory model. Finally, in the *Turing machine* regime, the particle acts like the moving 'head' of a Turing machine (hence, some questions about defect dynamics are formally undecidable).

Codimension. Let $\mathbb{A} \subset \mathbb{Z}^D$ be the 'admissible' part of $\mathbf{a}$, and let $\mathbf{A} \subset \mathbb{R}^D$ be the union of all closed unit D-cubes around the points in $\mathbb{A}$. I say the defect in $\mathbf{a}$ is a *domain boundary* (or has *codimension one*) if $\mathbf{A}$ is disconnected; (e.g. if $D = 1$ then *all* defects are of this type). For any $d \in [2...D]$, I say $\mathbf{a}$ has a defect of *codimension* d if the homotopy group $\pi_{d-1}(\mathbf{A})$ is nontrivial; loosely speaking, this means that the defect has the topology of a $(D-d)$-dimensional object in $\mathbb{Z}^D$. (The precise definition actually involves an inverse limit of homotopy groups computed on larger and larger 'scales'; see ⟨**13**, §1.1⟩.)

Persistent and Essential Defects. Many defects are *persistent*: they are not destroyed by iteration of Φ. The question is: why not? I say a defect in $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}$ is *essential* if it cannot be removed simply by redefining $\mathbf{a}$ in a neighbourhood of the defective region. Essential defects reflect 'global structural properties' of $\mathbf{a}$, relative to the topological dynamical system $(\mathbf{X}, \Phi, \sigma)$. If $\Phi : \mathbf{X} \longrightarrow \mathbf{X}$ is bijective, then any essential defect is persistent ⟨**12**, Prop.1.6⟩. In publications ⟨**12, 13**⟩, I showed how many essential defects admit 'algebraic invariants', which arise from the spectral or cohomological structure of $\mathbf{X}$, and which are Φ-invariant.

Spectral Defects. The *spectral group* of $(\mathbf{X}, \Phi, \boldsymbol{\sigma})$ is the group $\mathbb{S}_{\mathbf{X}} \subset \mathbb{C}^{D+1}$ of all 'generalized eigenvalues' of Φ and $\boldsymbol{\sigma}$; it encodes any (quasi)periodic structure in $\mathbf{X}$. A domain boundary in $\mathbf{a}$ can then separate two regions which are 'out of phase' with respect to this periodic structure; in ⟨**12**, §3⟩, I called this a *dislocation*. To any dislocation, we can associate a *displacement*: an element in the dual group $\widehat{\mathbb{S}}_{\mathbf{X}}$. If $\Phi : \mathbf{X} \longrightarrow \mathbf{X}$ is surjective, then every dislocation is Φ-persistent, and its displacement is Φ-invariant ⟨**12**, Theorems 3.6 and 3.14⟩. If two dislocations collide, then we simply add their displacements together in $\widehat{\mathbb{S}}_{\mathbf{X}}$.

Cohomological Defects. If $(\mathcal{G}, \cdot)$ is a group, and $\mathbf{X} \subset \mathcal{A}^{\mathbb{Z}^D}$ is a subshift, then a ($\mathcal{G}$*-valued, dynamical*) *cocycle* on $\mathbf{X}$ is a function $C : \mathbf{X} \longrightarrow \mathcal{G}^{\mathbb{Z}^D}$ such that, for any $\mathbf{x} \in \mathbf{X}$ and $\mathsf{y}, \mathsf{z} \in \mathbb{Z}^D$, $C(\mathbf{x})_{\mathsf{y}+\mathsf{z}} = C\left[\sigma^{\mathsf{z}}(\mathbf{a})\right]_{\mathsf{y}} \cdot C[\mathbf{a}]_{\mathsf{z}}$; see e.g. [Sch98]. Cocycles admit a natural notion of 'cohomology', and the set $\mathcal{H}^1_{\text{dyn}}(\mathbf{X}, \mathcal{G})$ of cocycle cohomology classes is a group if $(\mathcal{G}, \cdot)$ is abelian. If $\mathbf{x} \in \mathbf{X}$, and ζ is any polygonal path in $\mathbb{Z}^D$, then I define a kind of $\mathcal{G}$-valued 'path integral' of $C(\mathbf{a})$ along ζ, whose value depends only on the endpoints of ζ ⟨**13**, Lemma 2.10⟩. In particular, the integral around any loop is trivial. If $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}$ has a codimension-two defect (i.e. $\pi_1(\mathbf{A})$ is nontrivial), then one can also define $C(\mathbf{a})[\zeta]$ along a loop ζ in $\mathbf{A}$. If $C(\mathbf{a})[\zeta] \neq e_{\mathcal{G}}$, then the defect in $\mathbf{a}$ is essential ⟨**13**, Thm.2.8⟩; I call this a *pole*. If ζ' is another loop homotopic to ζ in $\mathbf{A}$, then $C(\mathbf{a})[\zeta] = C(\mathbf{a})[\zeta']$; thus, $C(\mathbf{a})$ defines a homomorphism $c_{\mathbf{a}} : \pi_1(\mathbf{A}) \longrightarrow \mathcal{G}$ —the *residue* of the pole ⟨**13**, Prop.2.7⟩. If $\Phi(\mathbf{X}) \subseteq \mathbf{X}$, then Φ induces an endomorphism Φ_* on $\mathcal{H}^1_{\text{dyn}}(\mathbf{X}, \mathcal{G})$; if Φ_* is surjective, then every $\mathcal{G}$-pole is Φ-persistent and its residue evolves under Φ in a well-defined way ⟨**13**, Prop.2.11⟩.

Next, I extended ideas of [CL90] and [GP95] to obtain algebreo-topological invariants for essential defects of *any* codimension. The space $\mathbb{R}^D$ admits a natural decomposition as a cubical cell complex with zero-skeleton $\mathbb{Z}^D$; see ⟨**13**, §3.1⟩. If $(\mathcal{G}, +)$ is an abelian group and $d \in [0...D)$, then a $\mathcal{G}$*-valued, d-dimensional invariant cocycle* on $\mathbf{X}$ is a function C which converts any element of $\mathbf{X}$ into a cocycle on this cell complex in a certain shift-invariant way ⟨**13**, §4.1⟩. If ζ is a *d-cycle* (i.e. a collection of d-cells in $\mathbb{R}^D$ with trivial boundary) and $\mathbf{x} \in \mathbf{X}$, then $C(\mathbf{x}, \zeta) = 0$. Suppose $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^D}$ has a defect of codimension $(d+1)$, and the nondefective region $\mathbf{A}$ in $\mathbf{a}$ admits a d-cycle ζ which does not bound any $(d+1)$-chain in $\mathbf{A}$. If $C(\mathbf{a}, \zeta) \neq 0$ then the defect in $\mathbf{a}$ is essential; we call this a *d-pole* ⟨**13**, Prop. 4.9⟩. Let $\mathcal{H}^d_{\text{inv}}(\mathbf{X}, \mathcal{G})$ be the abelian group of cohomology classes of d-dimensional invariant cocycles. If Φ is a CA and $\Phi(\mathbf{X}) \subseteq \mathbf{X}$, then Φ induces a natural endomorphism $\Phi_* : \mathcal{H}^d_{\text{inv}}(\mathbf{X}, \mathcal{G}) \longrightarrow \mathcal{H}^d_{\text{inv}}(\mathbf{X}, \mathcal{G})$. If Φ_* is surjective and certain technical conditions are satisfied, then any d-pole is Φ-persistent ⟨**13**, Theorem 4.10(b)⟩. This is a natural generalization of of the 'pole/residue' theory for codimension-two defects (described above), because there is a natural isomorphism $\mathcal{H}^1_{\text{dyn}}(\mathbf{X}, \mathcal{G}) \cong \mathcal{H}^1_{\text{inv}}(\mathbf{X}, \mathcal{G})$; see ⟨**13**, Theorem 4.11(b)⟩.

Another way to understand such 'cohomological defects' is through a topological construction I call the *tile complex*. For any set $\mathcal{W}$ of D-dimensional Wang tiles, one can construct a D-dimensional cubical cell-complex, where each cube corresponds to the placement of a particular $\mathcal{W}$-tile at a particular point in $\mathbb{Z}^D$. Two cells have adjoining faces if they reside at neighbouring points in $\mathbb{Z}^D$ *and* the respective Wang tiles 'match' along the relevant face. The resulting cellular complex $\mathbf{K}$ can be seen as the 'sheaf of all admissible tilings' of $\mathbb{R}^D$ by $\mathcal{W}$; there is a surjective cellular map $\Pi : \mathbf{K} \longrightarrow \mathbb{R}^D$, and the admissible $\mathcal{W}$-tilings of $\mathbb{R}^D$ correspond bijectively with the continuous sections of Π; see ⟨**13**, §3.3⟩.

Furthermore, if $\mathbf{w}$ is a 'partial' tiling (e.g. one with a codimension-two hole in it), then $\mathbf{w}$ defines a 'partial' section $\varsigma_{\mathbf{w}}$ of Π (on the complement of this hole). Any loop γ in $\mathbb{R}^D$ around the hole then lifts (via $\varsigma_{\mathbf{w}}$) to a loop $\widetilde{\gamma}$ in $\mathbf{K}$. If $\widetilde{\gamma}$ is *not* nullhomotopic in $\mathbf{K}$, then the hole in $\mathbf{w}$ is an essential defect. In this way, we can use the fundamental group $\pi_1(\mathbf{K})$ to assign algebraic invariants to codimension-two essential defects in $\mathbf{w}$; this is very closely related to the *tiling group* of [CL90]. Through a similar process, we can use the dth homotopy groups and/or (co)homology groups of $\mathbf{K}$ to attach algebraic invariants to codimension-$(d+1)$ defects in $\mathbf{w}$ $\langle$**13**, Theorem 3.7$\rangle$.

If $\mathbf{X} \subset \mathcal{A}^{\mathbb{Z}^D}$ is any SFT, then we can represent $\mathbf{X}$ using a set $\mathcal{W}_{\mathbb{B}}$ of Wang tiles corresponding to the elements of $\mathbf{X}_{\mathbb{B}}$ (where $\mathbb{B} \subset \mathbb{Z}^D$ is a large enough box); the dth homotopy and (co)homology groups of the tile complex of $\mathcal{W}_{\mathbb{B}}$ then encode information about codimension-$(d+1)$ defects in $\mathbf{X}$. However, to detect very large-scale constraints in $\mathbf{X}$, and to internalize the iteration of Φ, we must repeat this construction for a sequence of boxes $\mathbb{B}$ whose size tends to infinity, and then take suitable direct or inverse limits; this yields the *projective homotopy groups* and *projective (co)homology groups* of $\mathbf{X}$ $\langle$**13**, §3.5$\rangle$, which generalize the *projective fundamental group* introduced by [GP95].

If $\Phi : \mathcal{A}^{\mathbb{Z}^D} \longrightarrow \mathcal{A}^{\mathbb{Z}^D}$ is a CA and $\mathbf{X} \subset \mathcal{A}^{\mathbb{Z}^D}$ is a Φ-invariant SFT, then Φ induces group endomorphisms of the projective homotopy and (co)homology groups of $\mathbf{X}$ $\langle$**13**, Proposition 3.5$\rangle$. If these homomorphisms are injective or surjective, then defects with nontrivial homotopic/(co)homological invariants are Φ-persistent $\langle$**13**, Corollary 3.8$\rangle$.

(ii) Asymptotic Randomization in Algebraic CA. If $(\mathcal{A}, +)$ is a finite abelian group, then $\mathcal{A}^{\mathbb{Z}^D}$ is a compact abelian group under pointwise addition. We say $\Phi : \mathcal{A}^{\mathbb{Z}^D} \longrightarrow \mathcal{A}^{\mathbb{Z}^D}$ is a *linear cellular automaton* (LCA) if Φ is also a group endomorphism of $\mathcal{A}^{\mathbb{Z}^D}$ (equivalently, the local rule ϕ is a homomorphism from $\mathcal{A}^{\mathbb{V}}$ into $\mathcal{A}$). The Haar measure on $\mathcal{A}^{\mathbb{Z}^D}$ is the uniform Bernoulli measure η (the maximal entropy measure on $\mathcal{A}^{\mathbb{Z}^D}$). If μ is some other probability measure on $\mathcal{A}^{\mathbb{Z}^D}$, then Φ *asymptotically randomizes* μ if $\text{wk}^*\lim_{\mathbb{J} \ni j \to \infty} \Phi^j(\mu) = \eta$ for some $\mathbb{J} \subseteq \mathbb{N}$ of Cesàro density 1.

For $\mathcal{A} = \mathbb{Z}_{/p}$ (p prime), [Miy79, Lin84, CL93, MM98, FMMN00] had earlier shown that certain simple one-dimensional LCA on $\mathcal{A}^{\mathbb{Z}}$ asymptotically randomized Bernoulli and Markov measures with full support on $\mathcal{A}^{\mathbb{Z}}$. In $\langle$**25**, Theorems 12 & 15$\rangle$, Reem Yassawi and I showed: if $\mathcal{A}$ is *any* finite abelian group, then for *any* $D \geq 1$, almost *any* 'nontrivial' D-dimensional LCA asymptotically randomizes any measure on $\mathcal{A}^{\mathbb{Z}^D}$ whose Fourier coefficients satisfy a decay condition we called *harmonic mixing*. This includes: all 'nontrivial' Bernoulli measures on $\mathcal{A}^{\mathbb{Z}^D}$ $\langle$**25**, Prop.6 & 7$\rangle$, and all Markov random fields on $\mathcal{A}^{\mathbb{Z}^D}$ with full support $\langle$**19**, Thm.15$\rangle$ or 'local freedom' $\langle$**15**, Thm.1.3$\rangle$. In publication $\langle$**23**$\rangle$, I extended this to the case when $(\mathcal{A}, \cdot)$ is a nilpotent nonabelian group. In publication $\langle$**15**$\rangle$ we extended this to measures supported on sofic subshifts. In publications $\langle$**14, 16**$\rangle$ (joint work with Alejandro Maass and Servet Martinez), we obtained analogous results if μ is supported on a Φ-invariant *subgroup shift* (a subshift $\mathbf{G} \subset \mathcal{A}^{\mathbb{Z}^D}$ which is also a subgroup); in this case Φ randomizes μ to a maximum-entropy (often Haar) measure on $\mathbf{G}$.

(iii) Measure Rigidity in Algebraic CA. A CA exhibits 'measure rigidity' if the uniform Bernoulli measure η is the only invariant measure satisfying certain 'nondegeneracy' conditions. For example, if $\mathcal{A} = \mathbb{Z}_{/p}$ (p prime) and Φ is a radius-1 LCA on $\mathcal{A}^{\mathbb{Z}}$, then [HMM03] proved that the only Φ-ergodic, positive-entropy, (Φ, σ)-invariant measure is η. I proved similar rigidity results in the case when $(\mathcal{A}, \cdot)$ is a nonabelian group and Φ is a multiplicative CA ⟨**18**, Corol.3.4⟩, as well as when $(\mathcal{A}^{\mathbb{Z}}, *)$ is a (possibly nonabelian) group shift and Φ is a CA and endomorphism of $\mathcal{A}^{\mathbb{Z}}$ ⟨**18**, Thm.5.2⟩. This work was then extended by [Sab07]. In paper ⟨**7**⟩ I showed: if $\mathcal{R}$ is a prime-characteristic ring, and $\mathcal{A}$ is an $\mathcal{R}$-module, and Φ is an $\mathcal{R}$-linear CA, then the only Φ-invariant measures on $\mathcal{A}^{\mathbb{Z}^D}$ satisfying a certain kind of multiple mixing are Haar measures on cosets of $\mathcal{R}$-submodule shifts of $\mathcal{A}^{\mathbb{Z}^D}$. Under certain conditions (e.g. $\mathcal{A} = \mathbb{Z}_{/p}$), the only such measure is η ⟨**7**, Corol.5⟩. These results complement the 'asymptotic randomization' results of **(ii)**.

(iv) CA vs. Highly Ordered Symbolic Systems. A *quasisturmian* subshift of $\mathcal{A}^{\mathbb{Z}^D}$ is a symbolic coding of a rigid $\mathbb{Z}^D$-action on a torus $\mathbb{T}^k$. A *Toeplitz* shift is defined by an infinite ascending hierarchy of periodic spatial structures along arithmetic progressions, while a *substitution* shift is defined by a self-similarity property which recurs on arbitrarily large scales. Unlike SFTs, these subshifts are highly 'ordered' structures (e.g. quasisturmian and substitution shifts have zero topological entropy, as do many Toeplitz shifts).

Toeplitz and substitution systems can be represented using *Bratteli-Vershik* (or *adic*) systems, which (like *odometers*) apply a 'successor' map to a lexicographically ordered space of sequences. In the paper ⟨**11**⟩, Ethan Coven, Reem Yassawi and I showed that, if $\Phi\colon \mathcal{A}^{\mathbb{Z}} \to \mathcal{A}^{\mathbb{Z}}$ is a left-permutative CA, then many Φ-orbit closures are topologically conjugate to odometers. Conversely, in paper ⟨**4**⟩, Reem and I showed how many adic transformations (including many odometers, Toeplitz, and substitution shifts) can be 'embedded' as orbit-closures in suitable CA. Finally, in publication ⟨**17**⟩, I studied the action of CA on quasisturmian systems. A CA Φ induces a continuous self-map Φ_* on the space $\mathfrak{P}$ of partitions of the torus $\mathbb{T}^k$. I showed there is a close correspondence between the properties of the topological dynamical system $(\mathfrak{P}, \Phi_*)$ and those of $(\mathcal{A}^{\mathbb{Z}^D}, \Phi)$.

(v) Symmetry Groupoids in Coupled Cell Networks. A *coupled cell network* (CCN) is a labelled digraph representing a multivariate ordinary differential equation (ODE). Each vertex (*cell*) corresponds to some state variables of the ODE; an arrow from cell i to cell j means that the evolution of i depends on the state of j. In publications ⟨**20, 21**⟩, we introduced a notion of 'local' symmetries between subcomponents of a CCN, which can induce patterns of synchrony between these components. Publication ⟨**21**⟩ was noted by *Thomson Scientific Essential Science Indicators* as 'one of the most cited recent papers in the field of Mathematics', and appeared as a 'Fast Moving Front' on the ESI Website in November, 2006 (`http://esi-topics.com`).

References

[BNR91] N. Boccara, J. Naser, and M. Roger. Particle-like structures and their interactions in spatiotemporal patterns generated by one-dimensional deterministic cellular automata. *Physical Review A*, 44(2):866–875, 1991.

[CH92] James P. Crutchfield and James E. Hanson. The attractor-basin portrait of a cellular automaton. *J. Statist. Phys.*, 66(5-6):1415–1462, 1992.

[CH93a] James P. Crutchfield and James E. Hanson. Attractor vicinity decay for a cellular automaton. *Chaos*, 3(2):215–224, 1993.

[CH93b] James P. Crutchfield and James E. Hanson. Turbulent pattern bases for cellular automata. *Phys. D*, 69(3-4):279–301, 1993.

[CH97] James P. Crutchfield and James E. Hanson. Computational mechanics of cellular automata: an example. *Phys. D*, 103(1-4):169–189, 1997. Lattice dynamics (Paris, 1995).

[CHS01] James P. Crutchfield, Wim Hordijk, and Cosma Rohilla Shalizi. Upper bound on the products of particle interactions in cellular automata. *Phys. D*, 154(3-4):240–258, 2001.

[CL90] J. H. Conway and J. C. Lagarias. Tiling with polyominoes and combinatorial group theory. *J. Combin. Theory Ser. A*, 53(2):183–208, 1990.

[CL93] Haiyan Cai and Xiaolong Luo. Laws of large numbers for a cellular automaton. *Ann. Probab.*, 21(3):1413–1426, 1993.

[DL10] Franz Dietrich and Christian List. Majority voting on restricted domains. *Journal of Economic Theory*, 145(2):512–543, 2010.

[Elo93a] Kari Eloranta. Partially permutive cellular automata. *Nonlinearity*, 6(6):1009–1023, 1993.

[Elo93b] Kari Eloranta. Random walks in cellular automata. *Nonlinearity*, 6(6):1025–1036, 1993.

[Elo94] Kari Eloranta. The dynamics of defect ensembles in one-dimensional cellular automata. *J. Statist. Phys.*, 76(5-6):1377–1398, 1994.

[FMMN00] Pablo A. Ferrari, Alejandro Maass, Servet Martínez, and Peter Ney. Cesàro mean distribution of group automata starting from measures with summable decay. *Ergodic Theory Dynam. Systems*, 20(6):1657–1670, 2000.

[GP95] William Geller and James Propp. The projective fundamental group of a $\mathbb{Z}^2$-shift. *Ergodic Theory Dynam. Systems*, 15(6):1091–1118, 1995.

[Gra84] P. Grassberger. Chaos and diffusion in deterministic cellular automata. *Phys. D*, 10(1-2):52–58, 1984. Cellular automata (Los Alamos, N.M., 1983).

[Har55] John C. Harsanyi. Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *Journal of Political Economy*, 63:309–321, 1955.

[Hed69] G. Hedlund. Endomorphisms and automorphisms of the shift dynamical systems. *Mathematical Systems Theory*, 3:320–375, 1969.

[HMM03] Bernard Host, Alejandro Maass, and Servet Martínez. Uniform Bernoulli measure in dynamics of permutative cellular automata with algebraic local rules. *Discrete Contin. Dyn. Syst.*, 9(6):1423–1446, 2003.

[KM00] Petr Kůrka and Alejandro Maass. Limit sets of cellular automata associated to probability measures. *J. Statist. Phys.*, 100(5-6):1031–1047, 2000.

[KM02] Petr Kůrka and Alejandro Maass. Stability of subshifts in cellular automata. *Fund. Inform.*, 52(1-3):143–155, 2002. Special issue on cellular automata.

[Kůr03] Petr Kůrka. Cellular automata with vanishing particles. *Fund. Inform.*, 58(3-4):203–221, 2003.

[Kůr05] Petr Kůrka. On the measure attractor of a cellular automaton. *Disc. & Cont. Dyn. Sys.*, (to appear), 2005.

[Lin84] D. A. Lind. Applications of ergodic theory and sofic systems to cellular automata. *Phys. D*, 10(1-2):36–44, 1984. Cellular automata (Los Alamos, N.M., 1983).

[LP02] C. List and P. Pettit. Aggregating sets of judgements: an impossibility result. *Economics and Philosophy*, 18:89–110, 2002.

[LP09] Christian List and Clemens Puppe. Judgement aggregation: a survey. In Paul Anand, Prasanta Pattanaik, and Clemens Puppe, editors, *Oxford handbook of rational and social choice*, chapter 19, pages 457–481. Oxford University Press, Oxford, UK., 2009.

[Miy79] Munemi Miyamoto. An equilibrium state for a one-dimensional life game. *J. Math. Kyoto Univ.*, 19(3):525–540, 1979.

[MM98] Alejandro Maass and Servet Martínez. On Cesàro limit distribution of a class of permutative cellular automata. *J. Statist. Phys.*, 90(1-2):435–452, 1998.

[Mye95] Roger B. Myerson. Axiomatic derivation of scoring rules without the ordering assumption. *Soc. Choice Welf.*, 12(1):59–74, 1995.

[Sab07] Mathieu Sablik. Measure rigidity for algebraic bipermutative cellular automata. *Ergodic Theory Dynam. Systems*, 27, 2007.

[Sch98] Klaus Schmidt. Tilings, fundamental cocycles and fundamental groups of symbolic $\mathbf{Z}^d$-actions. *Ergodic Theory Dynam. Systems*, 18(6):1473–1525, 1998.

[YL78] H. P. Young and A. Levenglick. A consistent extension of Condorcet's election principle. *SIAM J. Appl. Math.*, 35(2):285–300, 1978.

Teaching experience

TRENT UNIVERSITY:

Winter 2013	Math 1100	*Single-variable calculus* (continued from fall).
	Math 4450H	*Voting, Bargaining, and Social Choice.*
Fall 2012	Math 1100	*Single-variable calculus.*
	Math 3610H	*Discrete Optimization.*
	Math-Phys 3150H	*Partial differential Equations.*
July 2010 - July 2012		Research leave
Winter 2010	Math 1100	*Single-variable calculus* (continued from fall).
	Math 4951H	*Voting, Bargaining, and Social Choice.*
Fall 2009	Math 1100	*Single-variable calculus.*
	Math 3350H	*Linear Programming.*
	AMOD 5610H	*Foundations of Modelling.*
Winter 2009	Math 1100	*Single-variable calculus* (continued from fall).
	Math 3210H	*Mathematical Cryptography.*
	Math 4950H	*Game Theory.*
Fall 2008	Math 1100	*Single-variable calculus.*
	AMOD 5610H	*Foundations of Modelling.*
May-June 2008	Minicourse	*Cooperative Game Theory* (25 lecture hours, not for credit)
Winter 2008	Math 320H	*Number Theory.*
	Math 332H	*Groups & Symmetry.*
	Math 433H	*Algebraic Topology & Homological Algebra* (reading course).
Fall 2007	Math 220H	*Introduction to Pure Mathematics.*
Winter 2007	Math 310H	*Metric Spaces.*
	Math 472H	*Fractals and Complex Dynamics.*
One third‡ of	Math 492H	*Perspectives in Mathematics*
Fall 2006	Math 220H	*Introduction to Pure Mathematics.*
	Math 471H	*Chaos, Symbolic Dynamics, and Fractals.*
Winter 2006	Math 310H	*Metric Spaces.*
	Math 322	*Number Theory* (continued from fall).
	Math 426H	*Differential Geometry.*
	Math 497H	*Voting, Bargaining & Social Choice* (reading course).
Fall 2005	Math 220H	*Introduction to Pure Mathematics.*
	Math 322	*Number Theory.*
Winter 2005	(Research leave at Wesleyan University)	

Trent University teaching (continued)

Fall 2004	Math 220H	*Introduction to Pure Mathematics.*
	Math-Phys 305H	*Partial Differential Equations.*
	Math 306H	*Complex Analysis.*
	Math 406H	*Real Analysis & Measure Theory* (reading course)
Winter 2004	Math 306H	*Complex Analysis.*
	Math 330	*Algebra III: Groups, Rings & Fields* (continued from fall).
One sixth† of	Math 491H	*Perspectives in Mathematics*
Fall 2003	Math-Phys 305H	*Partial Differential Equations.*
	Math 330	*Algebra III: Groups, Rings & Fields.*
Winter 2003	Math 110	*Single-variable Calculus* (continued from fall).
	Math 330	*Algebra III: Groups, Rings & Fields* (continued from fall).
One third* of	Math 207H	*Introduction to numerical & computational methods.*
Fall 2002	Math 110	*Single-variable Calculus.*
	Math-Phys 305H	*Partial Differential Equations.*
	Math 330	*Algebra III: Groups, Rings & Fields.*

University of Houston:

2001-2002	Math 3363	*Introduction to Partial Differential Equations.*

University of Toronto:

Winter 2000	Math 233	*Second-Year Linear Algebra.*

(‡) Math 492H was a team-taught course, with three instructors, each of whom taught a four-week segment.
(†) Math 491H was a team-taught course, with six instructors, each of whom taught a two-week segment.

(∗) R. Yassawi and myself taught Math 207 from March 1 to April 30, after the original instructor took stress leave.

Teaching evaluations for 2002-2010 are available on the web at

`http://euclid.trentu.ca/pivato/evals.pdf`

Curriculum Development Activities. In 2004, I designed `Math 220H` (*Introduction to Pure Mathematics*), a new course to prepare Trent math majors to take advanced courses in the pure mathematics curriculum (e.g. abstract algebra, real analysis, topology). The `Math 220H` syllabus begins with basic set theory and proof techniques (*modus ponens*, induction, contradiction), and then introduces number theory (divisibility, primality, modular arithmetic), combinatorics, transfinite arithmetic, and elementary abstract algebra and topology. I taught `Math 220H` in 2004, 2005, 2006, and 2007. [`http://euclid.trentu.ca/220/`]

In the summer of 2006, I chaired a committee which implemented major reforms to the mathematics curriculum at Trent University, based mainly upon a draft proposal which I had prepared earlier in 2006. These reforms more efficiently allocated our limited teaching resources, while offering our students a more flexible program and a broader variety of advanced courses. We split, redesigned, rescheduled and renumbered many existing courses, and also created several new courses, including the following ones which I designed:

`MATH 285H` *The Mathematics of Art, Architecture and Music.*

`MATH 302H` *Differential Geometry.*

`MATH 321H` *Mathematical Cryptography.*

`MATH 433H` *Homological Algebra & Algebraic Topology.*
`MATH 435H` *Modules, Multilinear Algebra, & Linear Groups.*
`MATH 437H` *Commutative Algebra & Algebraic Geometry.*

Other teaching activities. In November 2007, I delivered a 3-hour presentation on *The mathematics of voting and elections*, to a class of Grade 9 & 10 students from Lindsay Collegiate Secondary School.

In May 2008, I delivered two full-day sessions on *The mathematics of voting and elections*, as part of Trent's *Mini Enrichment* 2008 (for Grade 7 & 8 students). [`http://euclid.trentu.ca/pivato/Teaching/math.voting.pdf`]

Remarks on Teaching

My job as a teacher is to help students teach themselves. But the student must be an active participant in the learning process. Lectures can provide a 'survey of the terrain', inspire a student's interest, and perhaps kindle some initial spark of understanding. But no one has ever achieved a full and deep understanding of any nontrivial mathematical concept from a lecture, however lucid. Full understanding only comes after a long process of autonomous study, contemplation, and exercise. My job is to guide the student through this process.

First, I must keep the student motivated and excited about the subject, because the journey is long and difficult, and the payoff, though real, is often remote and obscure. Thus, I try to make my lectures exciting, enthusiastic, and inspiring, and whenever possible I motivate the material with applications to other sciences, or connections to other parts of mathematics.

Second, I emphasize to the students that my role is really to *answer questions*, rather than simply recite from a script. Lectures should be interactive; otherwise they are just poor substitutes for a textbook. My lectures have an informal, conversational tone. I answer all questions carefully and completely, and maintain an affirming, friendly and welcoming environment, in the classroom, during my office hours, and in email communications.

Third, I explicitly discourage students from taking notes during lectures (unless they truly feel that this is a useful learning strategy). During lectures, the student should be free to listen, comprehend, and ask questions —not be distracted with the task of transcribing formulae from the blackboard. I emphasize to students that all of my lectures will closely follow the textbook (or my preprinted lecture notes). In the course outline which students receive on the first day of class, I provide a detailed lesson plan with 'recommended readings' from the textbook for every lecture of the entire semester; I strongly encourage students to read the recommended sections *prior* to each lecture, so that they will already have some familiarity with the material, and come prepared to ask questions. I cite (by number) each theorem or example from the text as I discuss it, so that students can easily cross-reference the classroom lectures with with the book. However, my lectures do not just recite the text verbatim; I focus on clarifying the more difficult concepts and arguments, and provide perspective, context, and methodological observations which go beyond the text.

Fourth, mathematics cannot be learned passively; it can only be learned through practice and active engagement. Thus, frequent evaluation is a critical part of my teaching strategy, not only to test the students, but more importantly, to make them learn the math by *doing* it. I give students weekly homework assignments or quizzes in all my courses. Rapid feedback is critical: after the deadline, I immediately post complete solutions to all questions on the course webpage, and I almost always grade and return these assignments within 48 hours of receipt, with careful comments explaining any errors. This makes the course an interactive learning experience, which keeps students motivated and improves their knowledge retention.

Fifth, I provide visual illustrations whenever possible. Many mathematical ideas are quite natural and visually intuitive, but they can easily be obscured by technicalities and cumbersome notation. Words and symbols are sometimes inadequate to describe images and motion. Thus, I illustrate concepts with multicolour chalk sketches during lectures, and I provide extensive illustrations in my preprinted notes. I work hard to find ways to visually illustrate ideas even in 'non-geometric' areas of mathematics (e.g. number theory, abstract algebra, measure theory). Sometimes, static, two-dimensional images are inadequate, so my course homepages contain links to full-colour, three-dimensional animations and interactive demonstrations. When teaching complex analysis, I used three-dimensional wire-and-paper models in the class to illustrate the Riemann surfaces of multifunctions and the stereographic projection onto the Riemann sphere.

Finally, I have developed several techniques to write mathematics more clearly through thoughtfully designed notation and carefully structured exposition. Computer scientists have long known that it is relatively easy to write software that *works*, but it is much more difficult (and important) to write working software that *other people* can understand. Similarly, it is relatively easy to write a formally correct proof; the real challenge is to make the proof easy to read. I borrow techniques from software design:

- Divide proofs into semi-independent modules ("subroutines"), each of which performs a simple, clearly-defined task.

- Integrate these modules together in an explicit hierarchical structure, so that their functional interdependence is clear from visual inspection.

- Explain formal steps with parenthetical heuristic remarks. For example, in a long string of (in)equalities, I often attach footnotes to each step, as follows:

  "$A \underset{(*)}{=} B \underset{(\dagger)}{\leq} C \underset{(\ddagger)}{<} D$. Here, (∗) is because [...]; (†) follows from [...], and (‡) is because [...]."

- Use suggestive notation, so that the visual appearance of a symbol acts as a mnemonic for its meaning. For example:

  – Employ different fonts (e.g. *italics*, **boldface**, sans serif, $\mathbb{M}$ATHBB, $\mathcal{MATHCAL}$, $\Gamma\rho\epsilon\epsilon\kappa$) for different classes of mathematical objects (e.g. numbers, vectors, linear operators, vector spaces, sets, functions, etc.) I explicitly introduce these font conventions, and then consistently adhere to them throughout the exposition.

  – Use letters from the same 'lexicographical family' to denote objects which 'belong' together. For example: If $\mathcal{S}$ and $\mathcal{T}$ are sets, then elements of $\mathcal{S}$ should be $s_1, s_2, s_3, \ldots$, while elements of $\mathcal{T}$ are $t_1, t_2, t_3, \ldots$. If $\mathbf{v}$ is a vector, then its entries should be $v_1, \ldots, v_N$. If $\mathbf{A}$ is a matrix, then its entries should be $a_{11}, \ldots, a_{NM}$.

  – Reserve upper-case letters (e.g. $J, K, L, M, N, \ldots$) for the bounds of intervals or indexing sets, and then use the corresponding lower-case letters (e.g. $j, k, l, m, n, \ldots$) as indexes. For example, $\forall n \in \{1, 2, \ldots, N\},\ A_n := \sum_{j=1}^{J} \sum_{k=1}^{K} a_{jk}^{n}$.

- Develop each concept at the appropriate level of generality and abstraction (*not* the maximum possible level of generality and abstraction).

- Proceed from the concrete to the abstract (not the other way around). Introduce examples early in the exposition, and return to them often. Illustrate the content of a theorem with one of these examples, *before* the statement of the theorem. (Then do more examples after the theorem).

These principles are illustrated in the textbook and preprinted lecture notes I have prepared for several of my courses at Trent University.

For **Partial Differential Equations** (`Math 3150H`), I wrote the textbook *Linear Partial Differential Equations and Fourier Theory*, (see page 1 of the CV for details). In addition to the illustrations in this text, I used `MAPLE` to generate 3D animations of many solutions to common PDEs. [`http://euclid.trentu.ca/pde/Animations`]

For **Voting, Bargaining, and Social Choice** (`Math 4951H`), I prepared *Voting, Arbitration, and Fair Division*: an introduction to the mathematical theory of social choice. This began as a short introduction to voting paradoxes and the impossibility theorems of Arrow, Sen, and Gibbard-Satterthwaite, and has grown into a 227-page package, with sections on voting power indices, bargaining theory, and fair division games. [`http://euclid.trentu.ca/pivato/Teaching/voting.pdf`].

For **Abstract Algebra** (`Math 3320H` and `3360H`), I prepared *Visual Abstract Algebra*: 245 pages of supplementary lecture notes, with approximately 40 figures. I also made many additional pictures available on the website. [`http://euclid.trentu.ca/Xaravve/330`]

It is especially important to reveal the 'hidden geometry' in abstract algebra, because it otherwise the subject quickly degenerates into meaningless formalism. Thus, I motivate groups as abstract 'spaces', or as devices which encode (geometric) symmetries. Likewise, I motivate ring theory via algebraic geometry (e.g. by introducing the Zariski topology), rather than simply treating it as 'generalized arithmetic'.

For **Real Analysis** (`Math 4790H`), I wrote *Analysis, Measure and Probability: A Visual Introduction.* As the title suggests, my strategy is to use extensive illustrations to explain the cumbersome technicalities which bog down most introductions to measure theory. This introduction is unconventional in two other ways as well. First, I develop the Lebesgue measure simultaneously with several other important examples (e.g. Lebesgue-Stieltjes, Hausdorff, Haar), treating all as special cases of the Carathéodory theory of outer measures. Second, I closely integrate applications (particularly probability theory and fractal geometry) with the abstract theory of measures. I believe that this keeps the student motivated and helps them better understand the meaning of the theory. [`http://euclid.trentu.ca/pivato/Teaching/measure.ps.gz`]

For **Single-variable calculus** (`Math 1100`), I prepared hand-written lecture notes for the entire course, with many colour illustrations. Using a document-projector, I made these notes the basis of all my lectures (in lieu of a blackboard). I also made all of them available to students on the course webpage. [`http://euclid.trentu.ca/Xaravve/1100/Notes`]