A fair pivotal mechanism for nonpecuniary public goods

2012 Public Choice Society meeting
Miami, Florida

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March 9, 2012

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- *Nonrivalrous* (my consumption does not impede your consumption).
- *Nonexcludable* (it is impossible to enforce private property rights).
- *Nonpecuniary* (they affect *subjective well-being*, rather than income).

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Thus, if alternative a is chosen and voter i pays a tax t_i, then i's utility will be $u_i(a) - c_i\, t_i$, where c_i is i's (constant) marginal utility of money.

1. Each voter i announces a monetary 'bid' $v_i(a)$ for each alternative a in $\mathcal{A}$ (thus, $v_i(a) - v_i(b)$ measures how much i prefers a over b).
2. We choose the alternative with the highest aggregate bid.
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If every voter deploys her dominant strategy, then the mechanism selects the a^* in $\mathcal{A}$ which maximizes the weighted utilitarian sum

$$\sum_{i \in \mathcal{I}} \frac{u_i(a)}{c_i}. \qquad (*)$$

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   It is more realistic to suppose the marginal utility of money is not constant, but *declining* for each voter (e.g. due to satiation).

2. **The Pivotal Mechanism is inequitable.**
   The political influence of voter i on the sum $\sum_{i \in \mathcal{I}} \frac{u_i(a)}{c_i}$ is proportional to $1/c_i$, which is (*ceteris paribus*) proportional to her income/wealth. Thus, rich voters have more influence than poor voters.
   For example, in 2007, 10% of Americans amassed nearly 50% of all income earned in the United States.
   Plausible assumption: people's bids in the pivotal mechanism are roughly proportional to their income.
   Thus, the richest 10% alone could effectively control the outcome. The pivotal mechanism would devolve into a plutocracy.

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"**Solution strategy:** Stratify voters by wealth. Examine the statistical distribution of voting behaviour within each wealth stratum. Now implement a 'wealth-adjusted' version of the pivotal mechanism, so that voters of all wealth strata exert the same influence, *on average.*"

To make this intuition precise, we must make several assumptions:

1. The population $\mathcal{I}$ of voters is large enough that we will have enough voters in each stratum to obtain good statistics.
2. There is not just one isolated referendum, but a series of many referenda on different issues.

   Thus, the statistics acquired from earlier referenda can be used to 'tune' the parameters of the mechanism for later referenda.
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Let $u_i^t : \mathcal{A}_t \longrightarrow \mathbb{R}_+$ be voter i's vNM utility function over $\mathcal{A}_t$ (for $t = 0, 1, 2, 3, \ldots$). Assume $\min_{a \in \mathcal{A}_t} u_i^t(a) = 0$.
We suppose i's joint vNM utility over $\mathcal{A}_t$ and wealth is *separable*.
Thus, if alternative a_t is chosen in referendum t, and voter i is left with a net wealth of w_i dollars, then her utility will be $u_i^t(a_t) + u_i^{\$}(w_i)$.

Stratification. Suppose $\mathcal{I} = \mathcal{I}_1 \sqcup \mathcal{I}_2 \sqcup \cdots \sqcup \mathcal{I}_N$, where, for each n in $[1 \ldots N]$, all voters in stratum $\mathcal{I}_n$ have roughly the same net wealth.
(**Example:** Let $N := 10$. Let $\mathcal{I}_n =$ the nth decile of wealth distribution.)
For all n in $[1 \ldots N]$, let $\varphi_n > 0$ be a positive 'fee'.
(*Heuristic:* the average marginal utility of φ_n dollars for voters in $\mathcal{I}_n$ should be roughly the average marginal utility of φ_m dollars for voters in $\mathcal{I}_m$).
We refer to the N-tuple $\varphi := (\varphi_1, \varphi_2, \ldots, \varphi_N)$ as the *fee schedule*.

Technical assumptions about the voters' utility functions

Imagine a series of referenda, occurring at times $t = 0, 1, 2, 3, \ldots$.
Let $\mathcal{A}_t$ be the menu of social alternatives for the referendum at time t.
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If enough voters hit the ceiling, then a^* might *not* maximize $\sum\limits_{i \in \mathcal{I}} \dfrac{u_i^t(a)}{c_i^t}$.

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How can we structure the fee schedule $\boldsymbol{\varphi} = (\varphi_1, \ldots, \varphi_N)$ to prevent this?

What is fair?

For all i in $\mathcal{I}$, let $V_i^t := \max_{a \in \mathcal{A}_t} v_i^t(a)$. (So $0 \leq V_i^t \leq 1$.)

Thus, V_i^t measures the *influence* of i on the outcome of referendum t.

Define $\overline{V}^t := \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} V_i^t$.

Thus, $\overline{V}^t$ is the *per capita average influence* of any voter on referendum t.

For all n in $[1 \ldots N]$, define $\overline{V}_n^t := \frac{1}{|\mathcal{I}_n|} \sum_{i \in \mathcal{I}_n} V_i^t$.

That is, $\overline{V}_n^t$ measures the *per capita average influence* of voters in wealth stratum n on referendum t.

We say that the fee schedule φ is *perfectly fair* in referendum t if:

- (F1) $V_i^t < 1$ for all voters i in $\mathcal{I}$ (i.e. *no voter hits the ceiling*); and
- (F2) $\overline{V}_n^t = \overline{V}^t$ for all n in $[1 \ldots N]$ —i.e. each stratum has the *same average influence* as every other stratum (rich or poor).

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(F2) $\overline{V}_n^t = \overline{V}^t$ for all n in $[1 \ldots N]$ —i.e. each stratum has the *same average influence* as every other stratum (rich or poor).

What is fair?

For all i in $\mathcal{I}$, let $V_i^t := \max_{a \in \mathcal{A}_t} v_i^t(a)$. (So $0 \leq V_i^t \leq 1$.)

Thus, V_i^t measures the *influence* of i on the outcome of referendum t.

Define $\overline{V}^t := \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} V_i^t$.

Thus, $\overline{V}^t$ is the *per capita average influence* of any voter on referendum t.

For all n in $[1 \ldots N]$, define $\overline{V}_n^t := \frac{1}{|\mathcal{I}_n|} \sum_{i \in \mathcal{I}_n} V_i^t$.

That is, $\overline{V}_n^t$ measures the *per capita average influence* of voters in wealth stratum n on referendum t.

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What is (p, ϵ)-fair?

Problem: It is generally impossible to guarantee that φ is perfectly fair.

Let $\epsilon > 0$ be some small but positive 'error tolerance'.

We say that the fee schedule φ is *ϵ-fair* in referendum t if

(F1$_\epsilon$) $\#\{i \in \mathcal{I};\ V_i^t = 1\} < \epsilon \cdot |\mathcal{I}|$ (i.e. *almost* nobody hit the ceiling).

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Problem: We can't even know whether φ was ϵ-fair until after the referendum has occurred. (We can't guarantee it in advance.)

Idea: If the statistical distribution of voting behaviour is roughly the same from one referendum to the next, then we can compute in advance the *probability* that φ will be ϵ-fair in any particular referendum.

Let $0 < p < 1$ and let $\epsilon > 0$.

Assume some fixed, known statistical distribution of voter behaviour.

The fee schedule φ is *(p, ϵ)-fair* if it has a probability of at least p to be ϵ-fair in any referendum where the behaviour of the voters is randomly generated according to this distribution.

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The calibration procedure

Idea: Use historical data to "tune" the fee schedule φ so it converges to (p, ϵ)-fairness over time. This is the second part of our mechanism.

Let $\varphi^t = (\varphi_1^t, \varphi_2^t, \ldots, \varphi_n^t)$ be the fee schedule at time t.

Fix a constant $\lambda > 1$ (calibration speed). Construct φ^{t+1} as follows:

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(R2) "Further adjust the fee of stratum n up (down) if the average influence of stratum n was higher (lower) than the population average."

Formally: Let $\overline{V}_n^{t,+} := \dfrac{1}{|\mathcal{I}_n^+|} \displaystyle\sum_{i \in \mathcal{I}_n^+} V_i^t$ and $\overline{V}_n^{t,-} := \dfrac{1}{|\mathcal{I}_n^-|} \displaystyle\sum_{i \in \mathcal{I}_n^-} V_i^t$.

Then define $s_n := \dfrac{\log(\overline{V}_n^{t,+}) - \log(\overline{V}_n^{t,-})}{\log(\varphi_n^{t,+}) - \log(\varphi_n^{t,-})} > 0$ $\left(\begin{smallmatrix}\text{Estimated elasticity of}\\ \text{average influence}\end{smallmatrix}\right)$.

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Idea: Use historical data to "tune" the fee schedule φ so it converges to (p, ϵ)-fairness over time. This is the second part of our mechanism.

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Theorem. *Suppose the population of voters is sufficiently large, and statistical distribution of their utility functions (over wealth and social alternatives) satisfies certain regularity assumptions. Then the calibration mechanism* (R1)-(R2) *will converge to a* (p, ϵ)*-fair fee schedule.*

Furthermore, the time to convergence is roughly $\mathcal{O}\left(\dfrac{\log(1/\epsilon)}{1-p}\right)$ *iterations.*

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Here, "certain regularity assumptions" means (roughly) that we treat the voters' utility functions as a set of independent random variables such that:

- All wealth strata have *same statistical distribution* of political preference intensities on any particular referendum.
- There is no correlation between a voter's political preference intensity and her utility function for wealth.
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Convergence of the calibration procedure (1)

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Meanwhile "sufficiently large" means

$$\text{Population} \quad \geq \quad \frac{8\sqrt{N^3+1}}{\epsilon\, C\, \sqrt{1-p}},$$

where N is the number of strata, and C is a constant (which depends on statistical distribution of the voters' preferences).

For example, if $N = 10$, $\epsilon = 0.01$, $p = 0.99$, and $C = 0.5$, then

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Formal analysis of convergence to fairness

Existence of a (p, ϵ)-fair fee schedule

To ensure that (p, ϵ)-fair fee schedule exists, we impose some 'regularity' conditions on the distribution of voter preferences. Let $\mathbb{N} := \{0, 1, 2, \ldots\}$.
For all t in $\mathbb{N}$, recall that $\mathcal{A}_t$ is the menu for referendum t.
For all i in $\mathcal{I}$, recall that $u_i^t : \mathcal{A}_t \longrightarrow \mathbb{R}$ is i's cardinal utility function.
Let $U_i^t := \max_{a \in \mathcal{A}_t} u_i^t(a)$. This measures the 'intensity' of voter i's preferences on referendum t. Here is our first assumption:

(U) For all t in $\mathbb{N}$, there is a probability distribution μ_t on $\mathbb{R}_+$ such that U_i^t is a μ_t-random variable, for all i in $\mathcal{I}$.
Also, $\{U_i^t;\ i \in \mathcal{I} \text{ and } t \in \mathbb{N}\}$ is a set of independent random variables.

Idea: All strata have *same statistical distribution* of political preference intensities on any particular referendum. *No correlation* of preference intensities between different referenda or between different voters.

Next, for all i in $\mathcal{I}$, and all $\varphi > 0$, let $C_i^t(\varphi) := u_i^\$(w_i^t) - u_i^\$(w_i^t - \varphi)$ be the 'cost' (in utility) of a fee of size φ for voter i at time t.
In particular, if voter i is in stratum $\mathcal{I}_n$, and deploys her dominant strategy for the mechanism (P1)-(P5), then $v_i^t(a) = \min\{1,\ u_i^t(a)/C_i^t(\varphi_i^t)\}$ for every alternative a in $\mathcal{A}_t$.

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For all n in $[1 \ldots N]$, assume a probability distribution ρ_n on $\mathcal{C}$ such that:

(C1) For every t in $\mathbb{N}$, the set $\{C_i^t\}_{i \in \mathcal{I}_n}$ is a set of independent, ρ_n-random elements of $\mathcal{C}$.

(C2) For every t in $\mathbb{N}$, and every i in $\mathcal{I}_n$, the random variables U_i^t and C_i^t are independent.

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Next, assume it is 'highly improbable' that a voter's political preference intensity will be huge, when measured in monetary terms. Formally:

(C3) For any $\epsilon > 0$, there is some constant $\overline{\varphi}_n^\epsilon > 0$ with the following property. For all t in $\mathbb{N}$, if U_t is a μ_t-random variable and C_n is an independent, ρ_n-random function, then $\mathrm{Prob}\left[U_t \geq C_n(\overline{\varphi}_n^\epsilon)\right] < \epsilon$.

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(C3) For any $\epsilon > 0$, there is some constant $\overline{\varphi}_n^\epsilon > 0$ with the following property. For all t in $\mathbb{N}$, if U_t is a μ_t-random variable and C_n is an independent, ρ_n-random function, then $\mathrm{Prob}\left[U_t \geq C_n(\overline{\varphi}_n^\epsilon)\right] < \epsilon$.

For example, suppose $\epsilon = 0.01$.

Then $\overline{\varphi}_n^\epsilon$ is the minimum fee required such that less than 1% of the voters in stratum $\mathcal{I}_n$ would be willing to pay more than $\overline{\varphi}_n^\epsilon$ dollars to change the outcome in a typical referendum.

Let $\mathcal{C}$ be the space of all nondecreasing functions from $\mathbb{R}_+$ to itself.
For all n in $[1 \ldots N]$, assume a probability distribution ρ_n on $\mathcal{C}$ such that:

(C1) For every t in $\mathbb{N}$, the set $\{C_i^t\}_{i \in \mathcal{I}_n}$ is a set of independent, ρ_n-random elements of $\mathcal{C}$.

(C2) For every t in $\mathbb{N}$, and every i in $\mathcal{I}_n$, the random variables U_i^t and C_i^t are independent.

This means there is *no correlation* between voters, or between a voter's wealth disutility and her political preference intensity.
Next, assume it is 'highly improbable' that a voter's political preference intensity will be huge, when measured in monetary terms. Formally:

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Then $\overline{\varphi}_n^\epsilon$ is the minimum fee required such that less than 1% of the voters in stratum $\mathcal{I}_n$ would be willing to pay more than $\overline{\varphi}_n^\epsilon$ dollars to change the outcome in a typical referendum.
(For a typical middle-class stratum, we would expect $\overline{\varphi}_n^{0.01}$ to be perhaps a few thousand dollars.)

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Finally, let $V_n(\varphi)$ be the *expected influence* which a random voter in stratum n would have on the outcome of referendum t, if $\varphi_n^t = \varphi$. We assume this function is well-behaved, and the same for all referenda.

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(C4) There is a decreasing, continuously twice-differentiable function $V_n : \mathbb{R}_+ \longrightarrow [0,1]$ such that $V(0) = 1$ and $\lim_{\varphi \to \infty} V(\varphi) = 0$, and such that for any $\varphi \geq 0$ and any t in $\mathbb{N}$, $V_n(\varphi)$ is the expected value of the random variable $\min\{1,\ U_t / C_n(\varphi)\}$, where U_t and C_n are as in (C3).

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Assumption **(C)** is the combination of assumptions (C1)-(C4).

Our first result: "*If the set $\mathcal{I}$ of voters is large enough, and we divide it into N equal-sized subgroups $\mathcal{I}_1, \ldots, \mathcal{I}_N$, then there is a (p, ϵ)-fair fee schedule.*"

Proposition 1. *Assume* (U) *and* (C). *Let $0 < V^* < 1$ be any constant.*

(a) *For all n in $[1 \ldots N]$, there is a unique φ_n^* in $\mathbb{R}_+$ with $V_n(\varphi_n^*) = V^*$.*

Now let $0 < \epsilon, p < 1$, and suppose that

$$\text{(A)} \quad |\mathcal{I}| \quad \geq \quad \frac{8\sqrt{N^3+1}}{\epsilon\, V^* \sqrt{1-p}}, \quad \textit{and} \quad \text{(B)} \quad |\mathcal{I}_1| = |\mathcal{I}_2| = \cdots = |\mathcal{I}_N| = \frac{|\mathcal{I}|}{N}.$$

(b) *There is a constant $K > 0$ such that, for any t in $\mathbb{N}$, if $|\varphi_n^t - \varphi_n^*| < K\,\epsilon$ for all n in $[1 \ldots N]$, then φ^t satisfies $(\mathrm{F2}_\epsilon)$ with probability $\geq p$.*

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(F2$_\epsilon$) "$1 - \epsilon < |\overline{V}_n^t / \overline{V}^t| < 1 + \epsilon$ for all n in $[1 \ldots N]$
(i.e. all strata have almost the same influence)."

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(c) *If V^* is small enough, then $\boldsymbol{\varphi}^t$ will also satisfy condition $(\mathrm{F1}_\epsilon)$ with probability p or higher.*

$(\mathrm{F1}_\epsilon)$ "$\#\{i \in \mathcal{I};\ \ V_i^t = 1\} < \epsilon \cdot |\mathcal{I}|$ (i.e. almost nobody hits the ceiling)."

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Example: if $N = 10$, $\epsilon = 0.01$, $p = 0.99$, and $V^* = 0.5$, then $|\mathcal{I}| \geq 507,000$ suffices to satisfy inequality (A) (this is the population of a medium-sized city).

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If V^* is small enough, and we define $\boldsymbol{\varphi}^* := (\varphi_1^*, \ldots, \varphi_N^*)$ as in Prop.1(a), then Prop.1(b,c) guarantees that the fee schedule $\boldsymbol{\varphi}^*$ is $(0.99, 0.01)$-fair.

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Problem: What value of V^* is 'small enough' in Prop.1(c)?

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Example: if $N = 10$, $\epsilon = 0.01$, $p = 0.99$, and $V^* = 0.5$, then $|\mathcal{I}| \geq 507,000$ suffices to satisfy inequality (A) (this is the population of a medium-sized city).
If V^* is small enough, and we define $\boldsymbol{\varphi}^* := (\varphi_1^*, \ldots, \varphi_N^*)$ as in Prop.1(a), then Prop.1(b,c) guarantees that the fee schedule $\boldsymbol{\varphi}^*$ is $(0.99, 0.01)$-fair.

Problem: What value of V^* is 'small enough' in Prop.1(c)? Also, to compute $\varphi_1^*, \ldots, \varphi_N^*$ in Prop.1(a), we must know exact structure of the probability distributions $\{\mu_t\}_{t=1}^{\infty}$ and $\rho_1, \ldots, \rho_N$ in assumptions (U) & (C).

Let $(\varphi_1^0, \ldots, \varphi_N^0)$ be initial fee schedule at time 0. Let $\overline{\varphi}_1^\epsilon, \ldots, \overline{\varphi}_N^\epsilon$ be as in assumption (C3). Let λ be the 'calibration speed' in rule (R1).

For any $\epsilon > 0$, define $L(\epsilon) \; := \; \dfrac{\max\{\log(\overline{\varphi}_n^\epsilon/\varphi_n^0)\}_{n=1}^N}{\log(\lambda)}$.

Behaviour of L depends on shape of distributions $\rho_1, \ldots, \rho_N$ and $\{\mu_t\}_{t=1}^\infty$ in assumptions (U) and (C). Typically, $L(\epsilon) \to \infty$ very slowly as $\epsilon \searrow 0$.

Example. Under reasonable hypotheses, $L(\epsilon) = \mathcal{O}(\log(1/\epsilon))$ as $\epsilon \searrow 0$.

Furthermore, $L(\epsilon)$ is 'small' if initial guess φ_n^0 was not too far from $\overline{\varphi}_n^\epsilon$.

Example. If $\lambda = 1.26$, and $\varphi_n^0 \geq \overline{\varphi}_n^\epsilon/4$ for all $n \in [1 \ldots N]$, then $L(\epsilon) \leq 6$.

Proposition 2. *Let* $0 < \epsilon, p < 1$. *Suppose* $|\mathcal{I}| > 1/\epsilon\sqrt{1-p}$. *If only* (R1) *is applied during each referendum, then there will almost surely come a time* T_p^ϵ *such that, for all* $t > T_p^\epsilon$, *condition* $(\mathrm{F1}_\epsilon)$ *is satisfied with probability* p *or higher. The expected value of the random variable* T_p^ϵ *is at most*

$$\tfrac{1}{1-p}\; L\left(\epsilon - \tfrac{1}{|\mathcal{I}|\sqrt{1-p}}\right).$$

Example. Let $\epsilon := 0.01$. Suppose we want $(\mathrm{F1}_\epsilon)$ to be violated in less than 4% of all referenda (i.e. $p := 0.96$). If $|\mathcal{I}| \geq 10\,000$ and $L(0.0095) \leq 6$, then 150 iterations of (R1) will usually suffice to guarantee this.

Let $(\varphi_1^0, \ldots, \varphi_N^0)$ be initial fee schedule at time 0. Let $\overline{\varphi}_1^\epsilon, \ldots, \overline{\varphi}_N^\epsilon$ be as in assumption (C3).

(C3) "For any $\epsilon > 0$, there is some constant $\overline{\varphi}_n^\epsilon > 0$ with the following property. For all t in $\mathbb{N}$, if U_t is a μ_t-random variable and C_n is an independent, ρ_n-random function, then $\mathrm{Prob}\left[U_t \geq C_n(\overline{\varphi}_n^\epsilon)\right] < \epsilon$."

(That is: it is "highly improbable" that a voter's political preference intensity will be huge, when measured in monetary terms.) Let λ be the 'calibration speed' in rule (R1).

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(R1) "Let $E_t := \#\{i \in \mathcal{I};\ \ V_i^t = 1\}/|\mathcal{I}|$ (i.e. fraction of voters hitting ceiling). If $E_t \geq \epsilon$, then set $\varphi_n' := \lambda \cdot (E_t/\epsilon) \cdot \varphi_n^t$, for all n in $[1 \ldots N]$. Otherwise, if $E_t < \epsilon$, then set $\varphi_n' := \varphi_n^t$, for all n in $[1 \ldots N]$."

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($\mathrm{F1}_\epsilon$) "$\#\{i \in \mathcal{I};\ V_i^t = 1\} < \epsilon \cdot |\mathcal{I}|$ (i.e. almost nobody hits the ceiling)". *The expected value of the random variable T_p^ϵ is at most*

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Recall that $\overline{V}_n^t := \dfrac{1}{|\mathcal{I}_n|}\displaystyle\sum_{i\in\mathcal{I}_n} V_i^t$. For any $\varphi > 0$, let $V_n(\varphi)$ be *expected influence* of stratum n on referendum t, if $\varphi_n^t = \varphi$, from Assumption (C4).

Lemma: *$\overline{V}_n^t = V_n(\varphi_n^t) + \gamma_n^t$, where γ_n^t is a random variable with mean zero, variance less than $1/|\mathcal{I}_n|$, and an "almost Gaussian" distribution.*

In practice, $|\mathcal{I}_n|$ is very large. Thus, $|\gamma_n^t|$ is very probably very small. **Example.** If $N = 10$ and each $\mathcal{I}_n$ represents one decile of the wealth distribution of 10 000 000 voters, then $|\gamma_n^t| < 0.004$, with probability greater than 99.99%. Thus, $\overline{V}_n^t \approx V_n^t(\varphi_n^t)$.
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(C4) "There is a decreasing, continuously twice-differentiable function $V_n : \mathbb{R}_+ \longrightarrow [0,1]$ such that $V(0) = 1$ and $\lim_{\varphi\to\infty} V(\varphi) = 0$, and such that for any $\varphi \geq 0$ and any t in $\mathbb{N}$, $V_n(\varphi)$ is the expected value of the random variable $\min\{1,\ U_t/C_n(\varphi)\}$, where U_t is a μ_t-random variable and C_n is an independent, ρ_n-random function."

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Proof sketch: Central Limit Theorem, plus assumptions (U) and (C). $\square$

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Proof sketch: Rule (R2) is actually the *Newton-Raphson* method for finding the values of $(\varphi^*_1, \ldots, \varphi^*_N)$ which we defined in Proposition 1(a). $\square$

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Conclusion. We have modified the Groves-Clarke pivotal mechanism to obtain a 'fair', strategy-proof implementation of weighted utilitarian social choice amongst *nonpecuniary* public goods.

The mechanism gives roughly equal influence to poor voters and rich voters.

Unresolved Problems:

- Most public goods have both pecuniary *and* nonpecuniary costs/benefits. (Example: law enforcement, urban zoning, roads, public education, commerce regulations, and the government itself.)
- The mechanism is very informationally intensive. But all votes must remain confidential, so that voters cannot be bribed or intimidated, or coordinate their actions in voting blocs.
- We assumed each voter's joint utility function over wealth and public goods was *separable*. But this is false; a large gain/loss of wealth will generally change a voter's preferences over public goods.
- The stochastic Clarke tax assumes voters are vNM expected utility maximizers. But this is empirically false (Kahneman & Tversky).
- The budget size must be fixed in advance, because otherwise the choice of public goods would involve an inextricable pecuniary component. How should society determine the size of this budget?

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Thank you.

These presentation slides are available at

`<http://euclid.trentu.ca/pivato/Research/pivotal.pdf>`

The paper is available at

`< http://mpra.ub.uni-muenchen.de/34525>`

Assumption (C)
Proposition 1
Proposition 2: The role of (R1)
Towards Proposition 3
Proposition 3

Conclusion