A statistical approach to epistemic democracy

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Epistemic social choice theory

Suppose a group of agents wants to determine the correct answer to some objective, factual question.

Assume all the agents have the same values or preferences.

The only conflict is over their beliefs about objective facts.

Question. Can the agents use some voting procedure reconcile their contradictory beliefs and arrive at the 'best' collective decision?

Examples:

- Condorcet's (1785) Jury Theorem: Under certain hypotheses, simple majority vote is a *maximum likelihood estimator* (MLE) when society must answer a yes/no question.
- H.P. Young (1986,1988,1995,1997): the Kemeny rule and the Borda rule are MLEs, when society faces a preference-aggregation problem involving more than two alternatives. (See also Conitzer and Sandholm (2005), Conitzer et al. (2009), Xia et al (2010), and Conitzer and Xia (2011).)

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We can distinguish between three sorts of questions:

1. A question about the best action or policy. (e.g: climate change).
   - Group wants to maximize value of a (universally agreed) utility function.
2. A question about facts, with some obvious background probability distribution over the possible answers (e.g. weather).
   - No utility function. 'Background knowledge' represented by a *prior probability distribution* over the possible answers.
3. A question about facts, with *no* obvious background probability distribution. (e.g. cause of Permian-Triassic extinction)
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Notation: Voting rules

- Let $\mathcal{I} := \{1, 2, 3, \ldots, N\}$, represent a set of voters.
- Let $\mathcal{V}$ be the set of signals which could be sent by each voter.
- A *profile* is a list $\mathbf{v} = (v_1, v_2, v_3, \ldots, v_N)$, which assigns a signal v_i in $\mathcal{V}$ to each voter i in $\mathcal{I}$.
- Let $\mathcal{V}^{\mathcal{I}}$ be the set of all profiles.
- Let $\mathcal{X}$ be the set of alternatives available to society (e.g. possible actions, possible answers to some question).
- A *voting rule* is a correspondence F from $\mathcal{V}^{\mathcal{I}}$ to $\mathcal{X}$
  For any profile $\mathbf{v}$ in $\mathcal{V}^{\mathcal{I}}$, we obtain a nonempty (usually singleton) subset $F(\mathbf{v}) \subseteq \mathcal{X}$.

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Let S be a function from $\mathcal{I} \times \mathcal{V} \times \mathcal{X}$ into $\mathbb{R}$.

For any $\mathbf{v}$ in $\mathcal{V}^{\mathcal{I}}$, let $F_S(\mathbf{v})$ be the set of all element(s) x in $\mathcal{X}$ which maximize the sum $S(1, v_1, x) + S(2, v_2, x) + \cdots + S(N, v_N, x)$.

This correspondence F_S from $\mathcal{V}^{\mathcal{I}}$ into $\mathcal{X}$ is the *scoring rule* defined by S.

Many common voting rules are scoring rules. For example:

- *Plurality rule*: $\mathcal{V} = \mathcal{X}$. $S(i, v, x) := \begin{cases} 1 & \text{if} \quad v = x; \\ 0 & \text{if} \quad v \neq x. \end{cases}$
- *Approval vote*: $\mathcal{V} := \{\text{all subsets of } \mathcal{X}\}$. $S(i, v, x) := \begin{cases} 1 & \text{if} \quad x \in v; \\ 0 & \text{if} \quad x \notin v. \end{cases}$
- *Range voting*: $\mathcal{V} := \{$ all functions mapping each element of $\mathcal{X}$ into a real number between 0 and 1$\}$. $S(i, v, x) := v(x)$.
- *Borda rule*: $\mathcal{V} := \{\text{all strict rankings of } \mathcal{X}\}$. $S(i, v, x) := r$ if x is ranked rth from the bottom according to the ranking v.
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Scoring rules: Definition and examples

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Scoring rules: Anonymity and vetos

- All of these scoring rules are *anonymous*: $S(i, v, x) = S(j, v, x)$ for all i and j in $\mathcal{I}$, all v in $\mathcal{V}$, and all x in $\mathcal{X}$.
- Thus, all voters have exactly the same "weight".
- A *non*-anonymous scoring rule is *Weighted plurality rule*. Now $\mathcal{V} = \mathcal{X}$, and $S(i, v, x) := \begin{cases} w_i & \text{if} \quad v = x; \\ 0 & \text{if} \quad v \neq x. \end{cases}$
  (Here, w_i is the *weight* of voter i).
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Maximum *a posteriori* estimators

Let $\mathcal{X}$ represent a set of possible "states of nature". The true state is unknown. Specify a prior probability distribution over $\mathcal{X}$.

Suppose that each voter has partial information about the true state, and this determines the way she votes.

For each i in $\mathcal{I}$ and x in $\mathcal{X}$, specify a probability distribution over $\mathcal{V}$ (the *error model*), which describes the sort of signal which voter i is likely to send if the true state of nature is x.

Assume that the signals of different voters are conditionally independent random variables, for any state of nature.

A *scenario* is a combination of a prior and an error model.

This scenario is *anonymous* if all voters are equally competent, and receive the same quantity and quality of information (i.e. for any x in $\mathcal{X}$, we have the same probability distribution on $\mathcal{V}$ for every i in $\mathcal{I}$).

If the conditional probability of voter i sending signal v is *zero*, given state of nature x, then the scenario says certain events are "impossible".

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MAP rationalizability

Given any scenario $\mathcal{C}$, and any profile $\mathbf{v}$ in $\mathcal{V}^{\mathcal{I}}$, we can use Bayes rule to compute the posterior distribution over $\mathcal{X}$, conditional on $\mathbf{v}$.

The *maximum a posteriori estimator* determined by $\mathcal{C}$ and $\mathbf{v}$ is the set $\mathrm{MAP}(\mathcal{C}, \mathbf{v}) := \{$element(s) of $\mathcal{X}$ which have maximal probability in this posterior distribution$\}$. (This is MLE if we use the uniform prior.)

A voting rule F is *MAP-rationalizable* if there exists some scenario $\mathcal{C}$ such that $F(\mathbf{v}) = \mathrm{MAP}(\mathcal{C}, \mathbf{v})$ for all $\mathbf{v}$ in $\mathcal{V}^{\mathcal{I}}$.

F is *anonymously MAP-rationalizable* if the scenario $\mathcal{C}$ is anonymous.

Theorem 1 (Pivato, 2011): *A voting rule is MAP-rationalizable if and only if it is a scoring rule.*

Furthermore, it is anonymously MAP-rationalizable if and only if it is an anonymous scoring rule.

Finally, it has no vetos if and only if the corresponding scenario has no impossibilities.

Example: The (weighted) plurality, Borda, Kemeny, approval, and range-voting rules are all MAP-rationalizable.

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Theorem 1 (Pivato, 2011): *A voting rule is MAP-rationalizable if and only if it is a scoring rule.*
Furthermore, it is anonymously MAP-rationalizable if and only if it is an anonymous scoring rule.
Finally, it has no vetos if and only if the corresponding scenario has no impossibilities.

Example: The (weighted) plurality, Borda, Kemeny, approval, and range-voting rules are all MAP-rationalizable.

MAP rationalizability

Given any scenario $\mathcal{C}$, and any profile **v** in $\mathcal{V}^{\mathcal{I}}$, we can use Bayes rule to compute the posterior distribution over $\mathcal{X}$, conditional on **v**.
The *maximum a posteriori estimator* determined by $\mathcal{C}$ and **v** is the set $\mathrm{MAP}(\mathcal{C}, \mathbf{v}) := \{$element(s) of $\mathcal{X}$ which have maximal probability in this posterior distribution$\}$. (This is MLE if we use the uniform prior.)
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The fact that voting rule F *can* be rationalized by some scenario $\mathcal{C}$ does not imply that $\mathcal{C}$ is a realistic description of the epistemic problem facing society.

What would a realistic description look like?

Often $\mathcal{X}$ has a sort of "geometry": some elements of $\mathcal{X}$ are "close together" (i.e. similar), while other elements are "far apart" (dissimilar). Suppose each voter makes a guess of the correct answer in $\mathcal{X}$ (i.e. $\mathcal{V} = \mathcal{X}$), and her guess is more likely to be *close* to the right answer than far away. Formally: there is a (decreasing) real-valued function E_i such that, if the right answer is x, then for any v in $\mathcal{X}$, the probability that voter i guesses v is given by $E_i[d(x, v)]$ (where $d(x, v) =$ "distance" between x and v). This is called a *metric* error model.

The scoring rule described in Theorem 1 is then a *metric* voting rule: for each i in $\mathcal{I}$ and x and v in $\mathcal{X}$, the value of $S(i, v, x)$ is a decreasing function of the distance between v and x.

The prior probability distribution on $\mathcal{X}$ translates into a "bias function" β, and the rule chooses all x in $\mathcal{X}$ which maximize the sum

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The median rule

Suppose the function E_i is *exponentially decaying*.
(i.e. $E_i(r) = a_i / b_i^r$ for some constants $a_i, b_i > 0$).
Then (for suitable prior probability distribution) the MAP is the *weighted median rule*. For any profile $\mathbf{v}$ in $\mathcal{V}^{\mathcal{I}}$, this rule picks the element(s) of $\mathcal{X}$ which *minimize* the sum

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where $w_1, w_2, \ldots, w_N$ are nonnegative "weights" assigned to the voters.

The rule is anonymous if and only if $a_1 = a_2 = \cdots = a_N$ and $b_1 = b_2 = \cdots = b_N$ (or equivalently, $w_1 = w_2 = \cdots = w_N = 1$). This yields the (unweighted) *median rule*, which minimizes $d(x, v_1) + \cdots + d(x, v_N)$.

The space $\mathcal{X}$ is called *homogeneous* if the geometry of $\mathcal{X}$ "looks the same" around each x in $\mathcal{X}$. (Example: a sphere or a plane.)
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Kemeny rule. Let $\mathcal{A}$ be a set of alternatives (e.g. presidential candidates)
Let $\mathcal{X} := \{$all preference relations over $\mathcal{A}\}$.
For any x and y in $\mathcal{X}$, let $d(x, y) := \#$ of pairs in $\mathcal{A}$ on which the orders of x and y disagree (this is the *Kendall metric*).
Then $(\mathcal{X}, d)$ is a homogeneous space.
The median rule on $\mathcal{X}$ is the *Kemeny rule*
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Committee selection. Let $\mathcal{A}$ be a set of possible candidates for some committee. Let $n \leq |\mathcal{A}|$.
Let $\mathcal{X} := \{$all committees comprised of exactly n candidates$\}$
For all x and y in $\mathcal{X}$, let $d(x, y) := \#$ of candidates on which x and y disagree.
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Examples of median rule

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Quasiutilitarian rules as expected utility maximizers

In a *quasiutilitarian* rule, each voter assigns a numerical "score" to each social alternative. She can give the same score to two or more alternatives (unlike Borda or (anti)plurality).

The rule chooses alternative with the highest average score. For example:

- *Classical utilitarianism* (CU) allows the score to be any real number.
- *Range voting* allows score to be any number between 0 and 1.
- *Approval voting* requires the score to be either 0 or 1.

Suppose that the score a voter gives to an alternative is her estimate of the "social utility" of that alternative. Each voter's estimates could be wrong. But under reasonable assumptions about the voters, one can show:

> *If the average score of alternative a is higher than the average score of alternative b, then the* conditionally expected utility *of a, given this information, is higher than the conditionally expected utility of b.*

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But there are some foundational problems with this entire program.

1. It begins with a familiar voting rule, and then "rationalizes" it with some probabilistic scenario, after the fact.
   But this is backwards. One should *begin* by specifying a prior probability distribution and an error model for the voters which captures the underlying epistemic problem as realistically as possible. *Then* compute the MLE/MAP/EUM for this model. This may or may not end up being a familiar voting rule.
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   But this is totally unrealistic. Voters come from similar cultural and educational backgrounds, draw upon the same public (mis)information, and communicate with each another. (Dietrich & Spiekerman, 2011). Also: the desire to conform or avoid conflict may lead to "herding" or "groupthink".

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   But even if all voters have same objectives, they may still have incentives to exaggerate, or suppress countervailing evidence, to counteract what they regard as the bias or misperceptions of the other voters (Austen-Smith & Bank, 1996, etc.)
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   But in most practical problems (e.g. recent debates over climate change and macroeconomics), this is not realistic.
   Some rationalizations (e.g. the Condorcet Jury Theorem, or my EUM-rationalization of quasiutilitarian rules) are fairly 'robust' to misspecification of the error model.
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Thank you.

These presentation slides are available at

<http://euclid.trentu.ca/pivato/Research/episteme2011.pdf>

The paper is available at

<http://euclid.trentu.ca/pivato/Research/stat_epist.pdf>

A longer and more technical version is available at

<http://mpra.ub.uni-muenchen.de/30292>