Emergent Defect Dynamics in Two-Dimensional Cellular Automata

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http://euclid.trentu.ca/Defect

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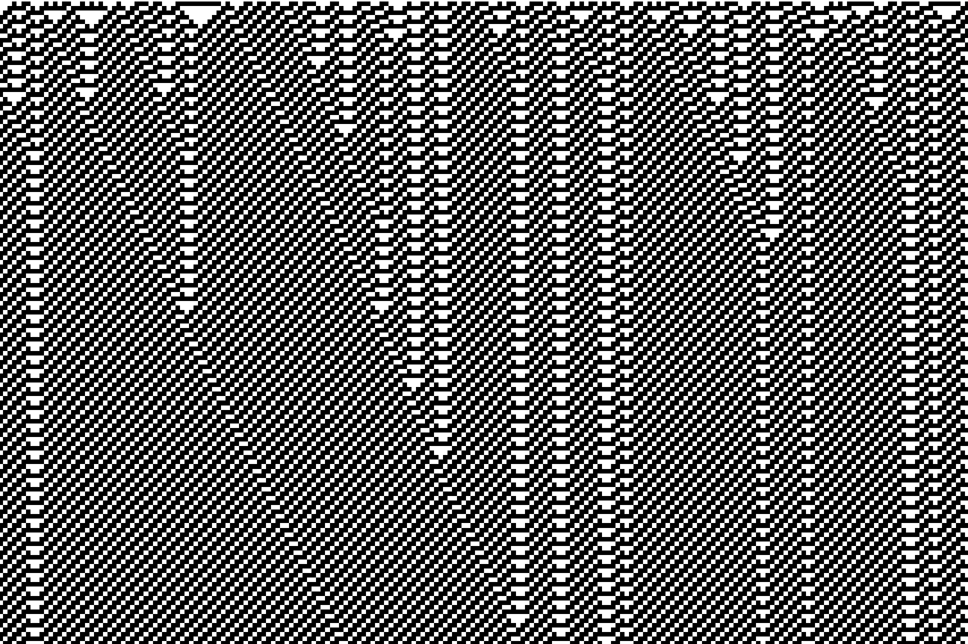
When 'defect particles' collide, they coalesce or annihilate according to some emergent 'defect chemistry'.

Emergent Defect Dynamics in ECA #62



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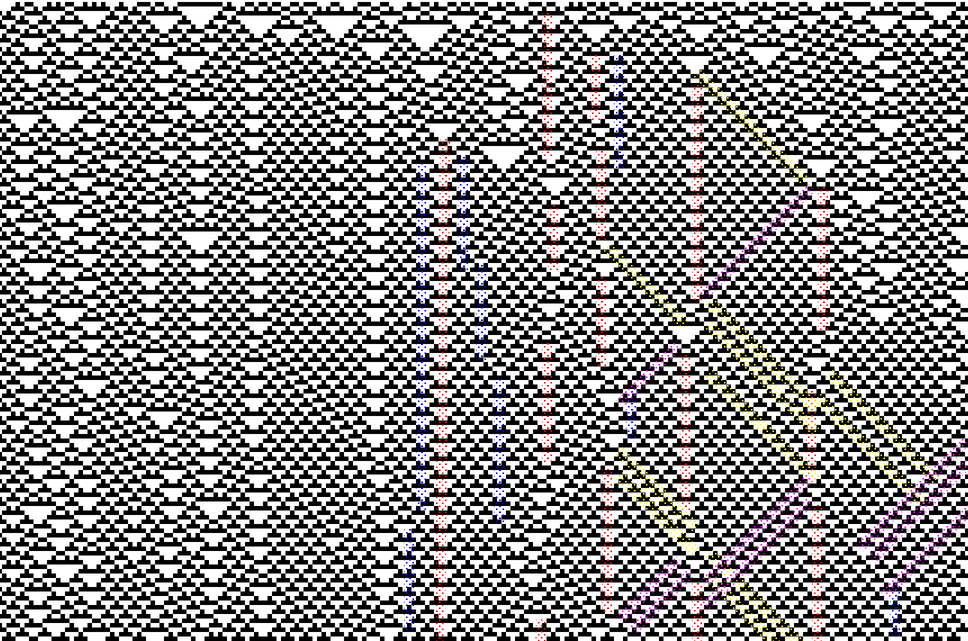

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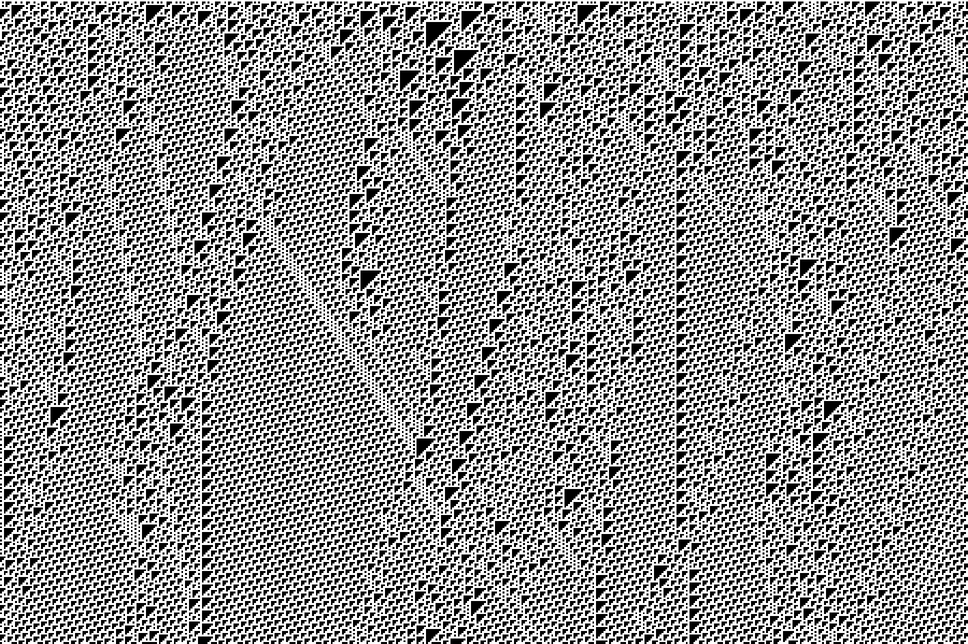

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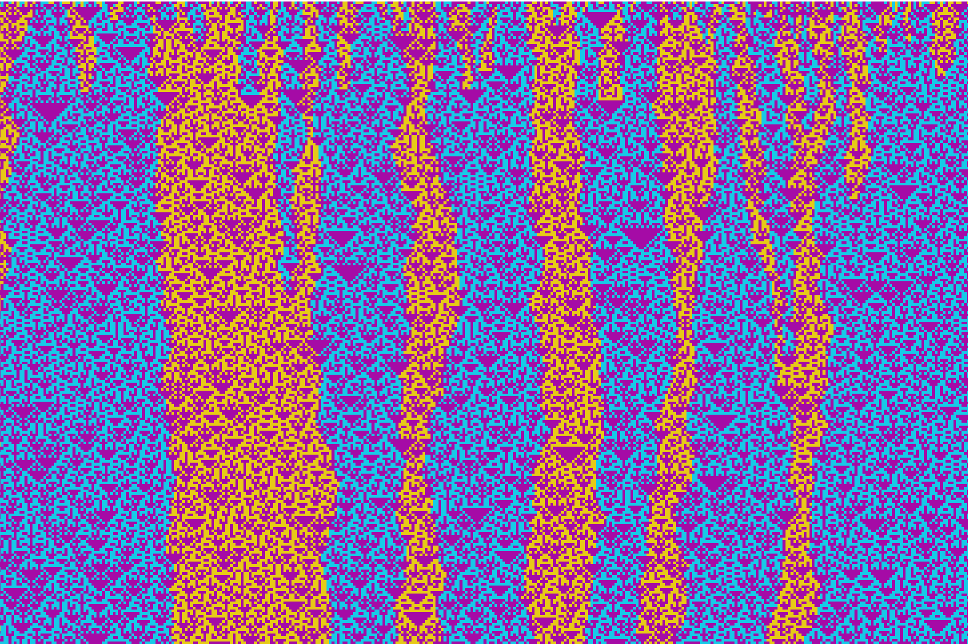

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  - *Turing* regime: particle acts like moving 'head' of Turing machine.

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If Φ 'respects' the underlying subshifts, then it must preserve these structural properties; hence the defects can neither be created nor destroyed, but only moved around and combined with other defects.

Interfaces

Let Φ be a CA, let $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^2}$, and let $\mathbf{b} := \Phi^{100}(\mathbf{a})$.
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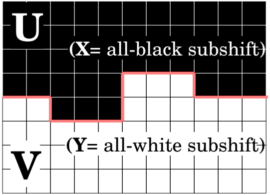
If $\mathbf{b}_{\mathbb{U}}$ and $\mathbf{b}_{\mathbb{V}}$ belong to disjoint subshifts $\mathbf{X}$ and $\mathbf{Y}$ then the boundary between them is called an *interface*.



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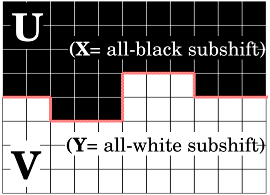

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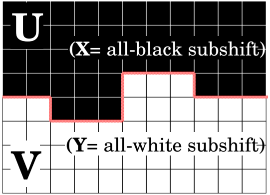

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Example: (ECA #184) Let $\mathcal{A} = \{\square, \blacksquare\}$. Let $\mathbf{X} := \{\ldots\blacksquare\blacksquare\blacksquare\ldots\}$, $\mathbf{Y} := \{\ldots.\square\square\square\ldots.\}$, and $\mathbf{Z} := \{\ldots\blacksquare\square\blacksquare\square\blacksquare\square\ldots.\}$. If Φ is ECA 184, then $\Phi(\mathbf{X}) = \mathbf{X}$, $\Phi(\mathbf{Y}) = \mathbf{Y}$, and $\Phi(\mathbf{Z}) = \mathbf{Z}$.

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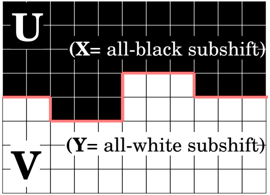

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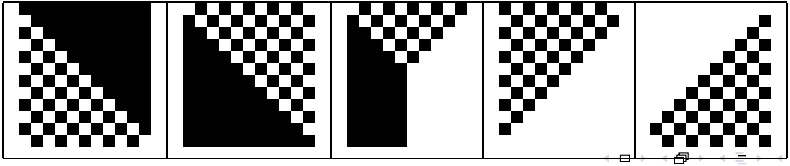

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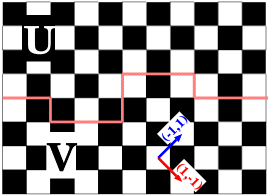

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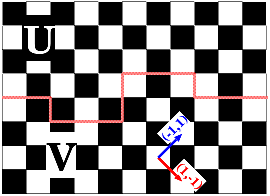

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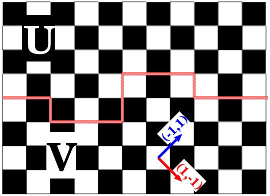

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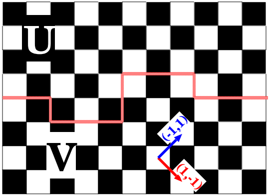

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Theorem: *If $\Phi : \mathbf{X} \longrightarrow \mathbf{X}$ is surjective, then any $\mathbf{X}$-dislocation persists under iteration of Φ, and its displacement is unchanging.* □[Fundamentae Informatica, 2007]

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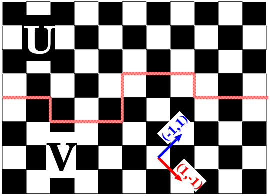

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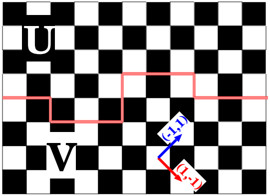
Theorem: *If $\Phi : \mathbf{X} \longrightarrow \mathbf{X}$ is surjective, then any $\mathbf{X}$-dislocation persists under iteration of Φ, and its displacement is unchanging.* □ [Fundamentae Informatica, 2007]

Example: (ECA#62) Let $\mathbf{X}$:= [. . . ■■□ ■■□ ■■□ . . .]. If Φ is ECA #62, then $\Phi|_{\mathbf{X}} = \sigma$, so $(\mathbf{X}, \Phi)$ is 3-periodic in both space and time, and $\Delta \cong \mathbb{Z}_{/3}$.

Dislocations

Let Φ be a CA, let $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^2}$, and let $\mathbf{b} := \Phi^{100}(\mathbf{a})$.
Let $\mathbb{U} \subset \mathbb{Z}^2$ and $\mathbb{V} \subset \mathbb{Z}^2$ be two 'regular domains' in $\mathbf{b}$.

Suppose $\mathbf{b}_{\mathbb{U}}$ and $\mathbf{b}_{\mathbb{V}}$ belong to the *same* subshift $\mathbf{X}$. Let $\mathbb{P} \subset \mathbb{Z}^2$ be a subgroup, and suppose $\mathbf{X}$ is $\mathbb{P}$-periodic. (i.e. $\forall\ \mathbf{x} \in \mathbf{X}$ and $\mathsf{p} \in \mathbb{P}$, $\sigma^{\mathsf{p}}(\mathbf{x}) = \mathbf{x}$.)
If $\mathbf{b}_{\mathbb{U}}$ and $\mathbf{b}_{\mathbb{V}}$ are 'out of phase' relative to this $\mathbb{P}$-periodic structure, then the boundary between $\mathbb{U}$ and $\mathbb{V}$ is called a *dislocation*.



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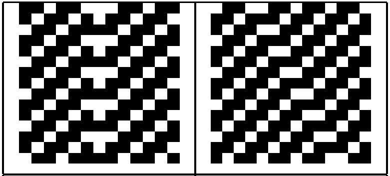

Disp=2	Disp=1

Height Functions & Gaps

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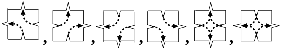
Example: (*Square Ice*) Let $\mathcal{I} := \{$  $\}$.

Let $\mathfrak{Ice} \subset \mathcal{I}^{\mathbb{Z}^2}$ be the subshift of all ‘tooth-in-groove’ tilings.

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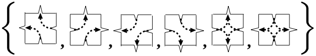
Example: (*Square Ice*) Let $\mathcal{I} :=$ .

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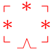
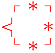
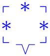
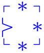
Define $h_1, h_2 : \mathcal{I} \longrightarrow \{\pm 1\}$ by $h_1($  $) := +1 =: h_2($  $)$ and

$h_1($  $) := -1 =: h_2($  $)$ ('$*$' means 'anything').

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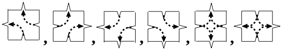
Example: (*Square Ice*) Let $\mathcal{I} := \{$  $\}$.

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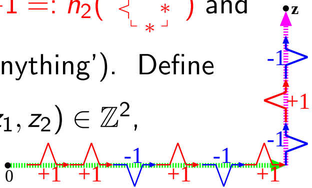

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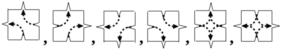
Example: (*Square Ice*) Let $\mathcal{I} := \{$  $\}$.

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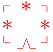
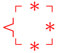
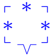
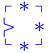
Define $h_1, h_2 : \mathcal{I} \longrightarrow \{\pm 1\}$ by $h_1($  $) := +1 =: h_2($  $)$ and $h_1($  $) := -1 =: h_2($  $)$ ('$*$' means 'anything'). Define $H : \mathbb{Z}^2 \times \mathfrak{Ice} \longrightarrow \mathbb{Z}$ so that, $\forall\, \mathbf{i} \in \mathfrak{Ice}$, $\forall\, z = (z_1, z_2) \in \mathbb{Z}^2$,

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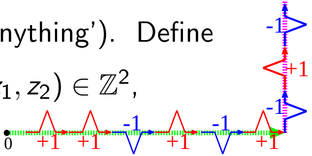

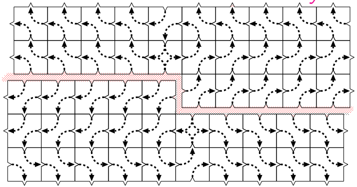

Using this height function, the domain boundary to the left can be visualized as....

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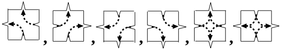
Example: (*Square Ice*) Let $\mathcal{I} :=$ {  }.

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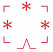
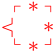
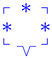
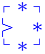
Define $h_1, h_2 : \mathcal{I} \longrightarrow \{\pm 1\}$ by $h_1($  $) := +1 =: h_2($  $)$ and $h_1($  $) := -1 =: h_2($  $)$ ('$*$' means 'anything'). Define $H : \mathbb{Z}^2 \times \mathfrak{Ice} \longrightarrow \mathbb{Z}$ so that, $\forall\ \mathbf{i} \in \mathfrak{Ice}$, $\forall\ \mathsf{z} = (z_1, z_2) \in \mathbb{Z}^2$,

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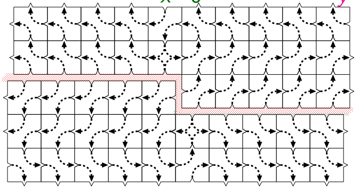
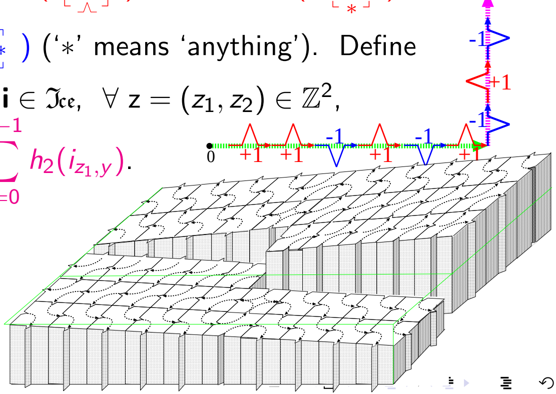
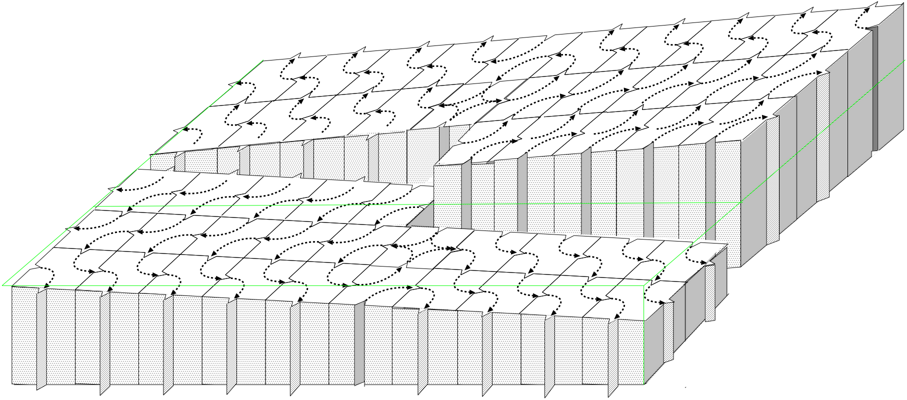




Gaps and Cohomology



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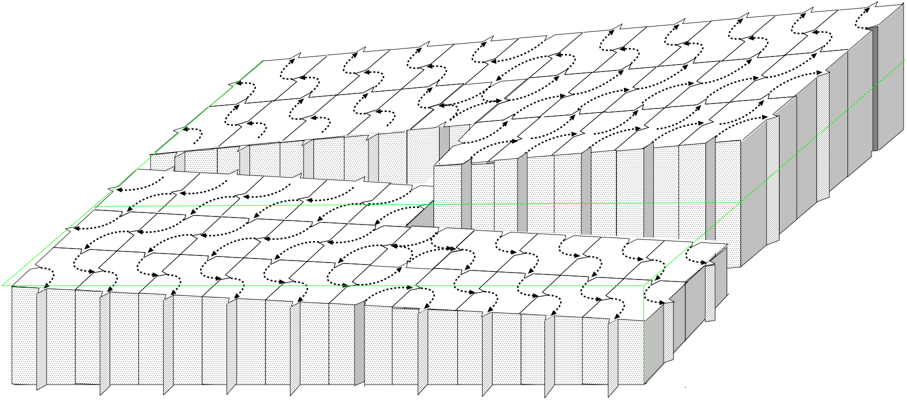

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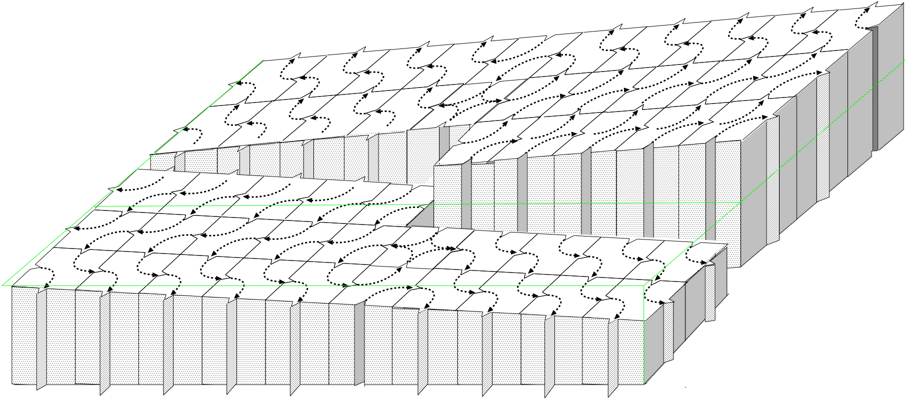

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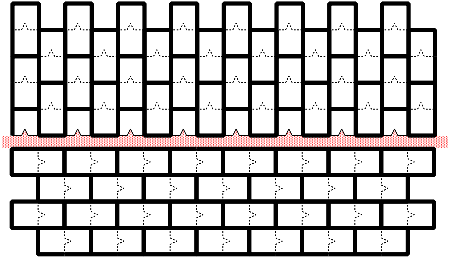
Theorem: *If Φ_* is surjective, then all gaps persist under Φ.* $\Box$[ErThDySy,2007]

Example: A gap in dominos



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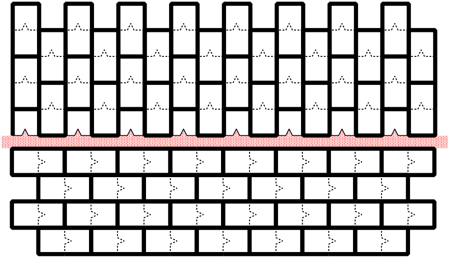

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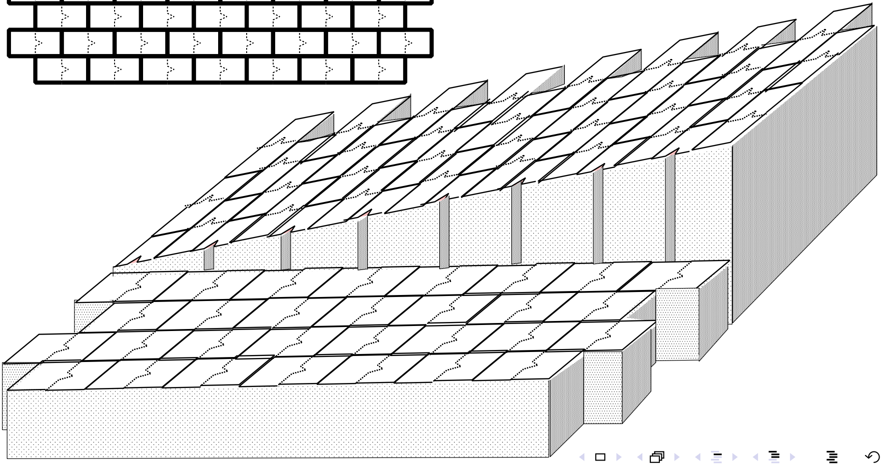

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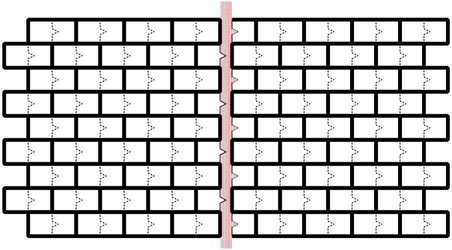

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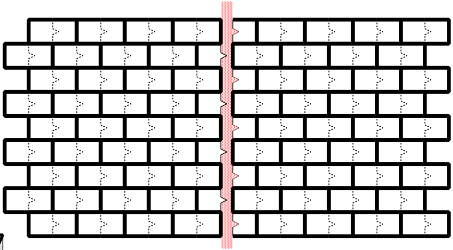

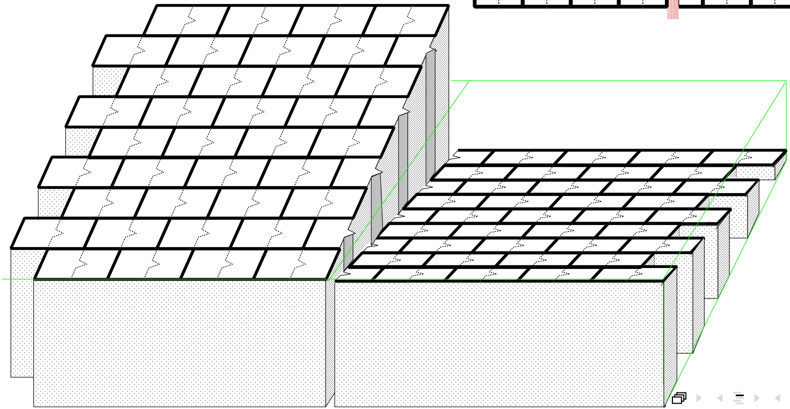

Codimension-two defects

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- However, some codimension-two defects still have a nontrivial cohomological signature, which renders them ‘indestructible’ under CA dynamics.

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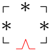
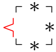
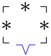
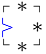
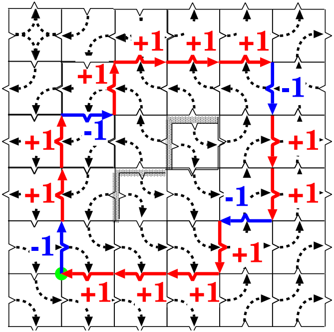
Example: Recall $H : \mathfrak{Ice} \times \mathbb{Z}^2 \longrightarrow \mathbb{Z}$

$h_1($  $) := +1 =: h_2($  $)$;

$h_1($  $) := -1 =: h_2($  $)$.

If γ is the clockwise trail around the defect, then $H(\gamma, \mathbf{x}) = 8$. Thus, $\mathbf{x}$ has a pole.



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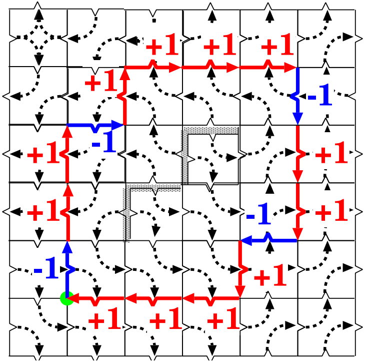

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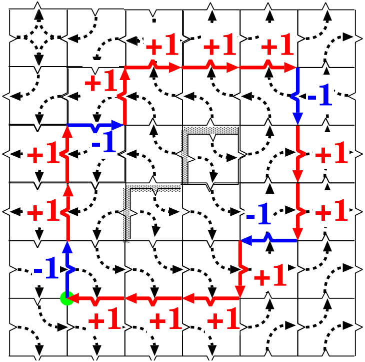

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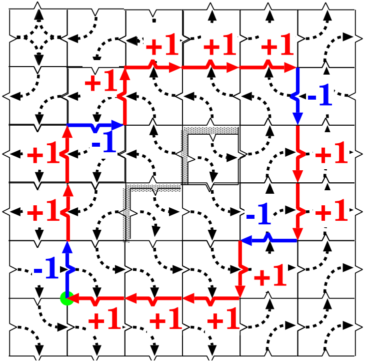

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Theorem: *If Φ_* is surjective, then all poles persist under Φ.* $\Box_{\text{[ErThDySy,2007]}}$

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4. If Φ is likely candidate, then look for domain boundaries, interfaces, and dislocations in $\mathbf{b}$.

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Let $k \in \mathbb{N}$. Let $\mathbb{K} := [0...k)^2 \subset \mathbb{Z}^2$. If $|\mathcal{A}| = 2$, then $|\mathcal{A}^{\mathbb{K}}| = 2^{k^2}$.

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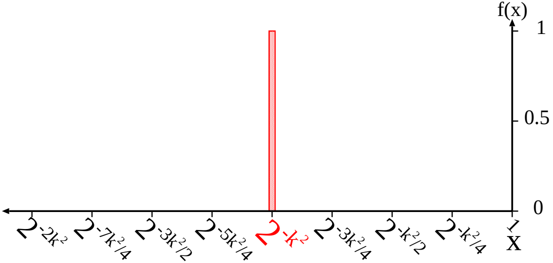

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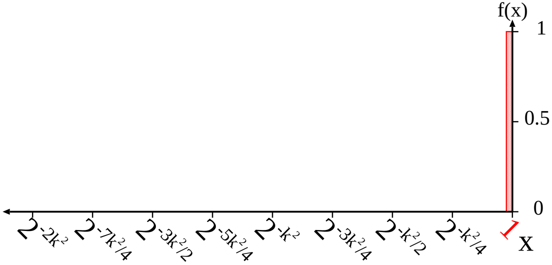

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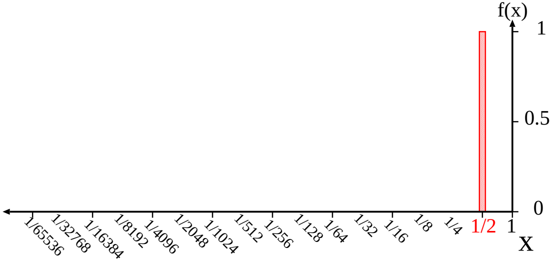

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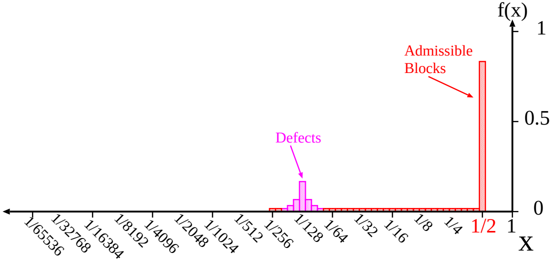

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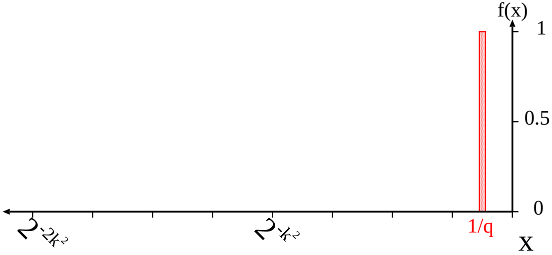

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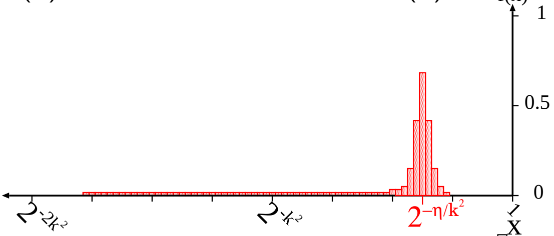

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Each $\mathbf{a} \in \mathcal{A}^{\mathbb{K}}$ has probability $p(\mathbf{a}) \in [0, 1]$.
Let M be large. For all $m \in [1...M]$ let

$$f(m) \;:=\; \frac{m}{M}\,\#\left\{\mathbf{a} \in \mathcal{A}^{\mathbb{K}}\,;\ \frac{m-1}{M} \le p(\mathbf{a}) \le \frac{m}{M}\right\}.$$

Example: *Subshift with defects.*
Should look like stationary measure on subshift, but with nonzero probability of inadmissible 'defective' blocks. Thus, $f(2^{-\eta k^2}) \approx 1$, and $f(x) = 0$ for all $x > 2^{-\eta k^2}$.

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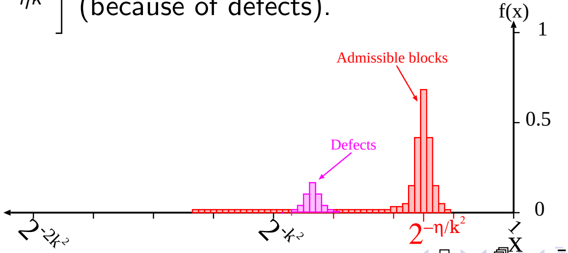

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Search should be conducted over the ‘simplest’ classes of 2-dimensional CA (analogous to ECA in one dimension). Thus, we set $\mathcal{A} = \{0, 1\}$.

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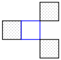
- *3-Cell nhood* 

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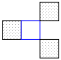
- *3-Cell nhood*  $2^{2^3} = 2^8 =$ 256 distinct local rules.

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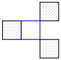
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.

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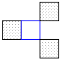
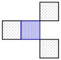
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle* 

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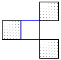
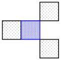
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion).

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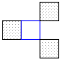
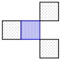
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion). Can exhaustively search entire space.

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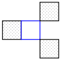
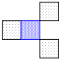
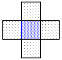
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion). Can exhaustively search entire space.
- *von Neumann* 

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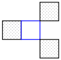
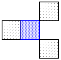
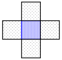
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion). Can exhaustively search entire space.
- *von Neumann*  $2^{2^5} = 2^{32} = 4\,294\,967\,296$ distinct local rules.

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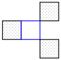
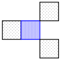
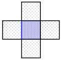
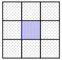
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion). Can exhaustively search entire space.
- *von Neumann*  $2^{2^5} = 2^{32} = 4\,294\,967\,296$ distinct local rules. Exhaustive search not feasible; instead we must randomly sample the rule space.
- *Moore* 

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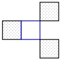
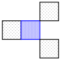
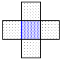
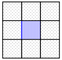
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion). Can exhaustively search entire space.
- *von Neumann*  $2^{2^5} = 2^{32} = 4\,294\,967\,296$ distinct local rules. Exhaustive search not feasible; instead we must randomly sample the rule space.
- *Moore*  $2^{2^9} = 2^{512} \approx 10^{154}$ distinct local rules.

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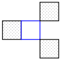
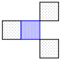
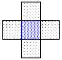
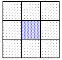
- *3-Cell nhood*  $2^{2^3} = 2^8 = 256$ distinct local rules. Can exhaustively search entire space.
- *Triangle*  $2^{2^4} = 2^{16} = 65536$ distinct local rules (32768 local rules modulo 0/1-inversion). Can exhaustively search entire space.
- *von Neumann*  $2^{2^5} = 2^{32} = 4\,294\,967\,296$ distinct local rules. Exhaustive search not feasible; instead we must randomly sample the rule space.
- *Moore*  $2^{2^9} = 2^{512} \approx 10^{154}$ distinct local rules. Exhaustive search not feasible; must randomly sample the space.

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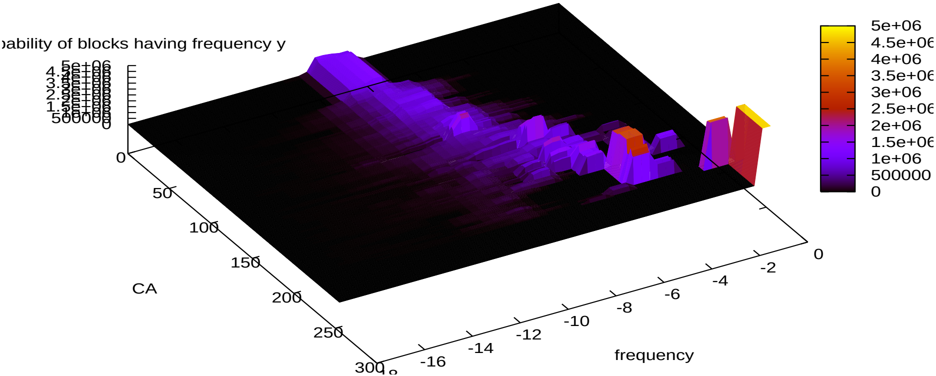

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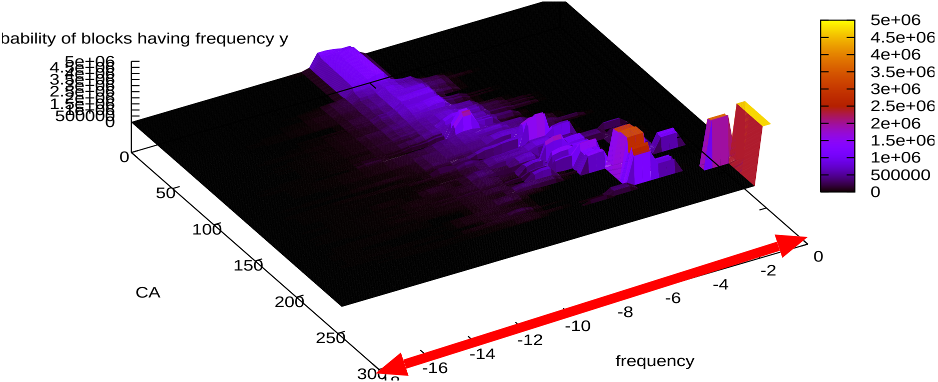

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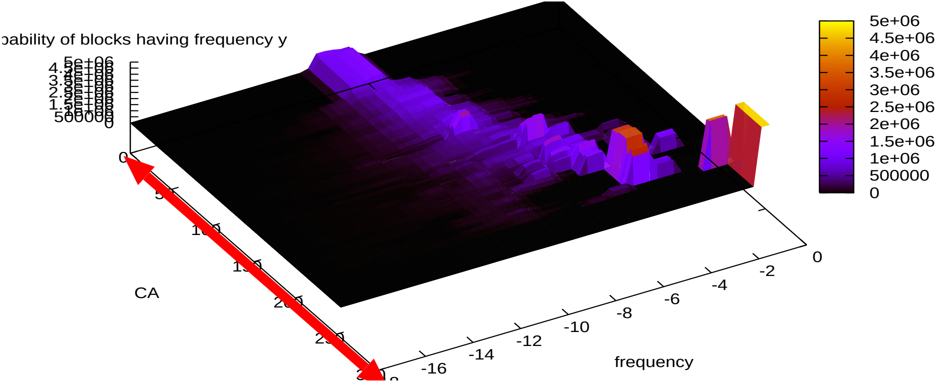

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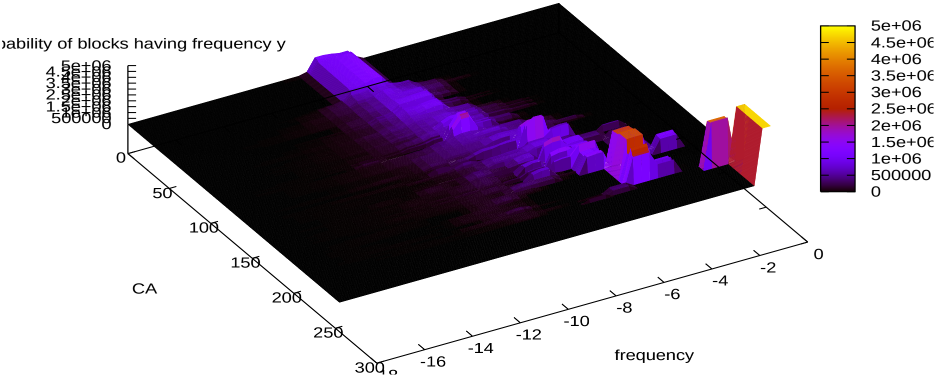

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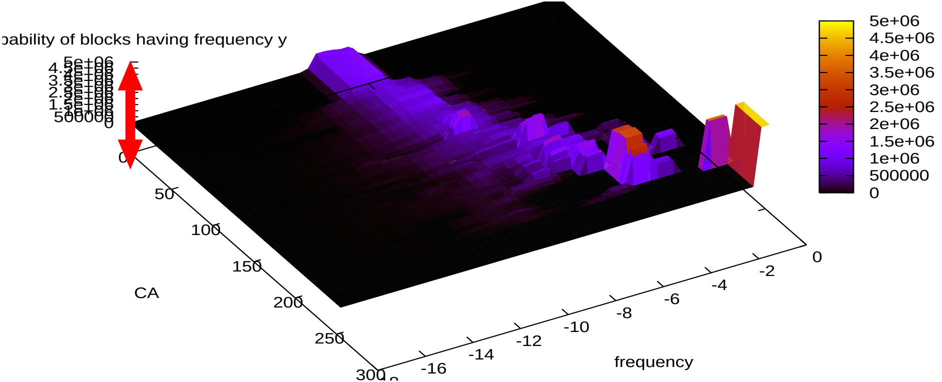

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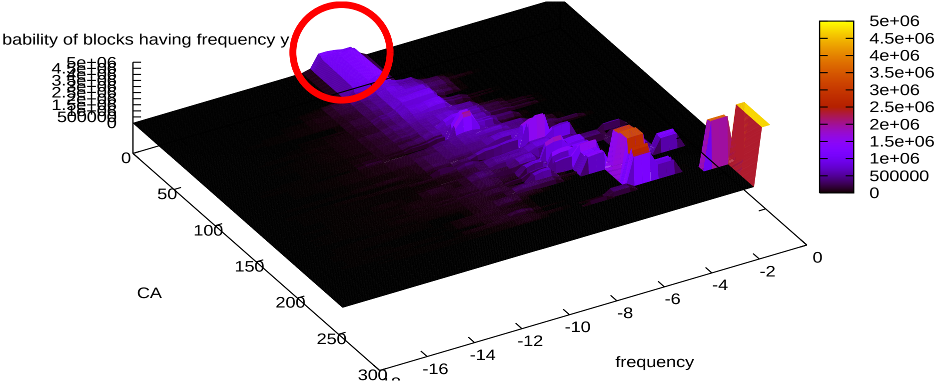

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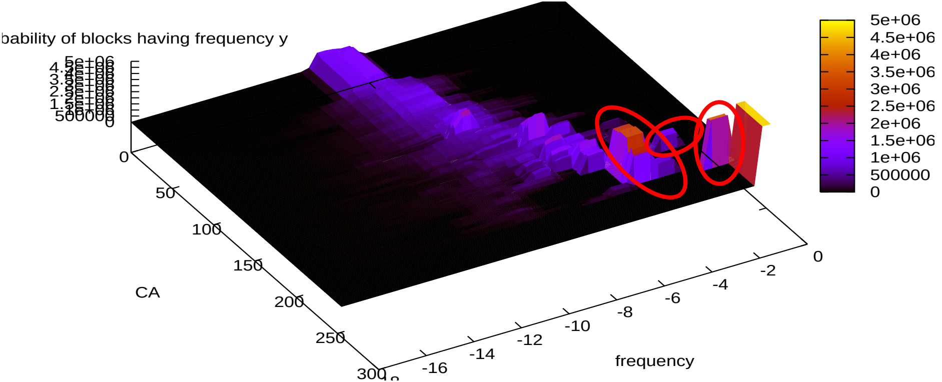

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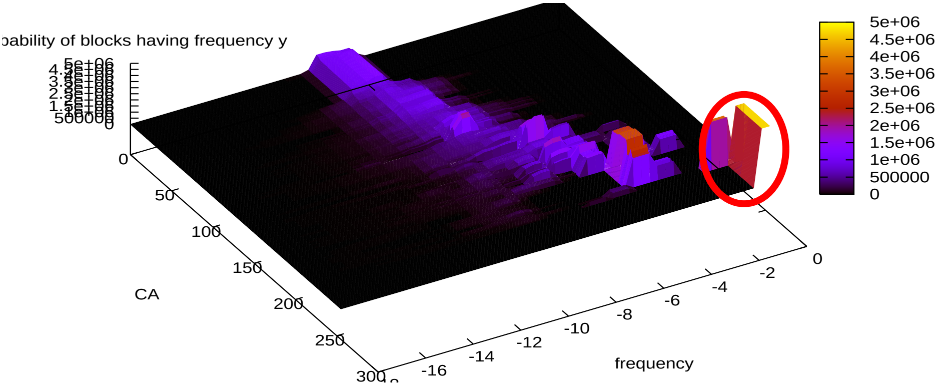

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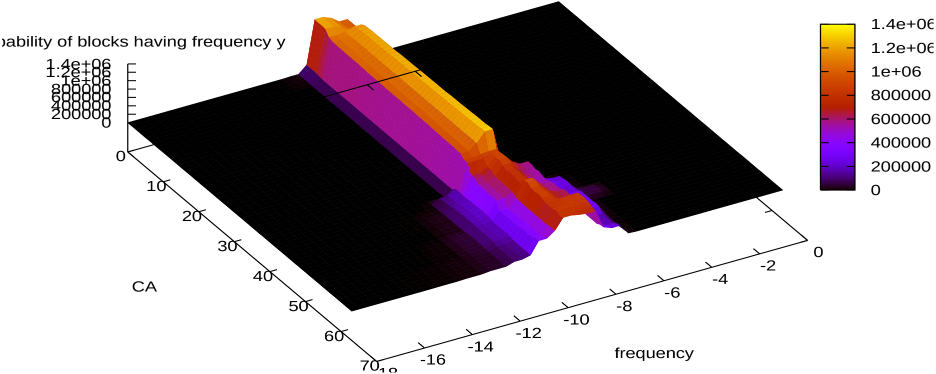
The red spike in right corner is caused by 'nilpotent' CA (where all initial conditions converge to the all-zero or all-one configurations).

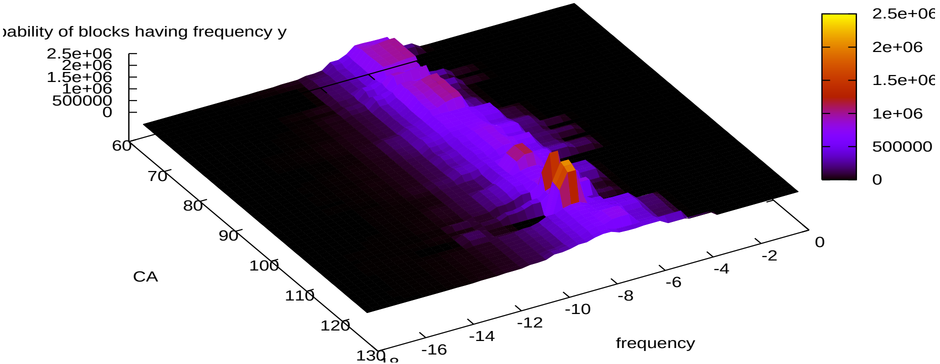
3-Cell CA landscape; Closeup 0-63



The red ridge (over $x = 2^{-9}$) is caused by CA which preserve the uniform measure.

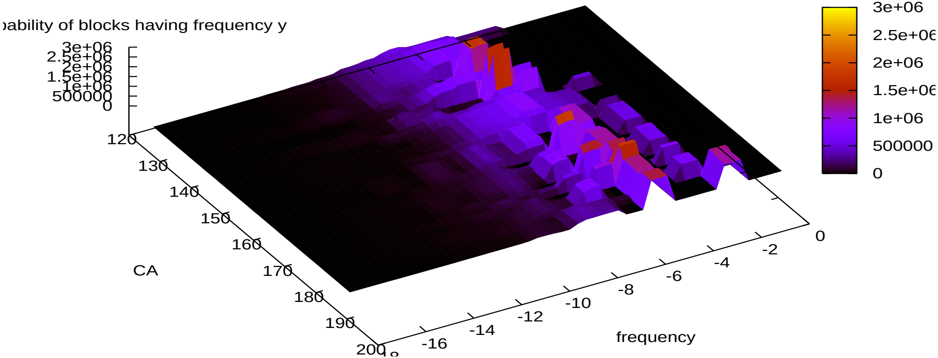
There are about 40 of these.

3-Cell CA landscape; Closeup 64-127



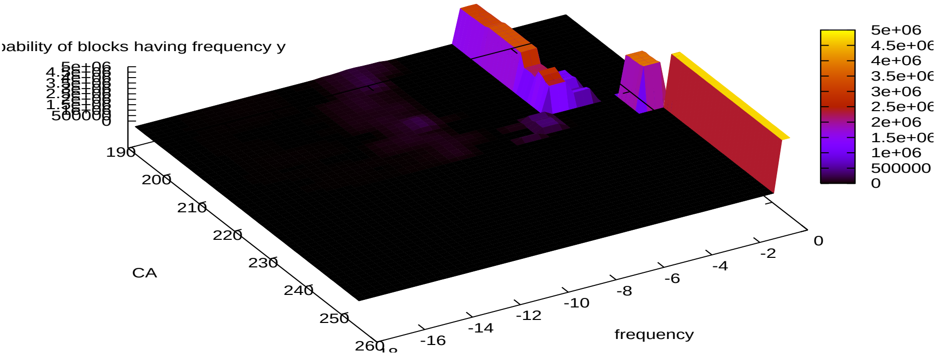
In this 'mountain range' region, the CA do not converge quickly to any low-entropy subshift; there is no indication of EDD.

3-Cell CA landscape; Closeup 128-191



The low ridges are caused by CA which begin to show the statistical signature of EDD.

3-Cell CA landscape; Closeup 192-255



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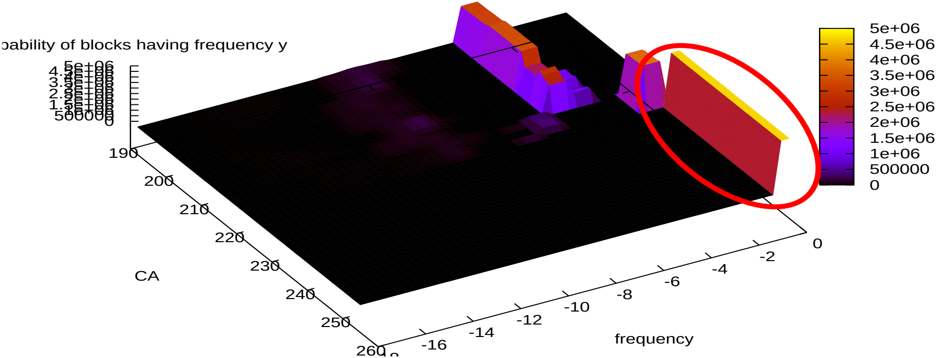
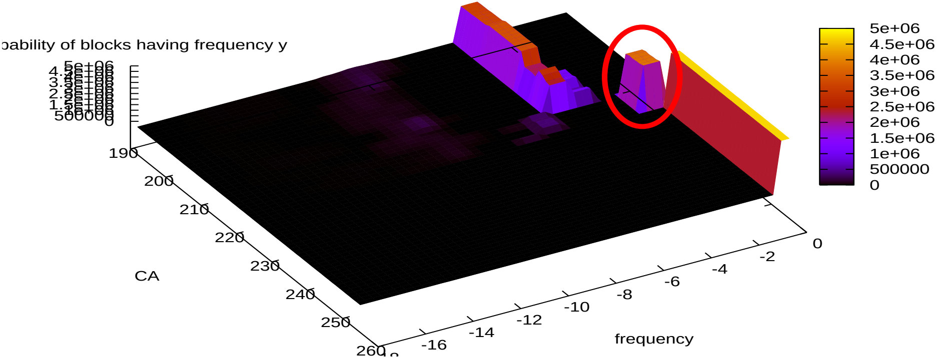


The red ‘wall’ in the right corner is caused by *nilpotent* CA, which converge to a constant (all-zeros or all-ones) configuration.

There are 46 of these.

3-Cell CA landscape; Closeup 192-255



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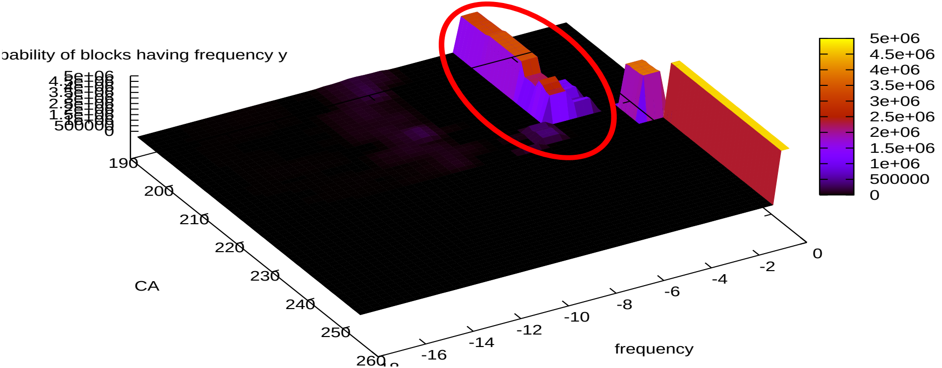
The purple massif next to the red wall is caused by CA whose background pattern has 2 elements (e.g. checkerboard, stripes).

There are 14 with stripes and 8 with checkerboard.

3-Cell CA landscape; Closeup 192-255



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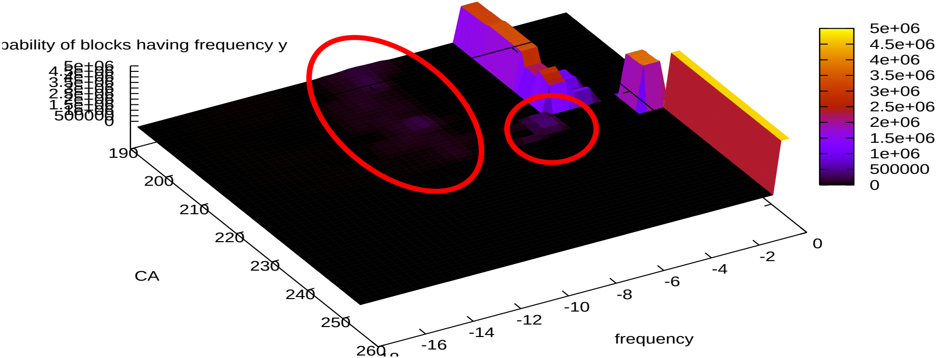
The red ridge is caused by CA which have regular background pattern(s) with 3-16 elements. There are at least 30 of these.

3-Cell CA landscape; Closeup 192-255



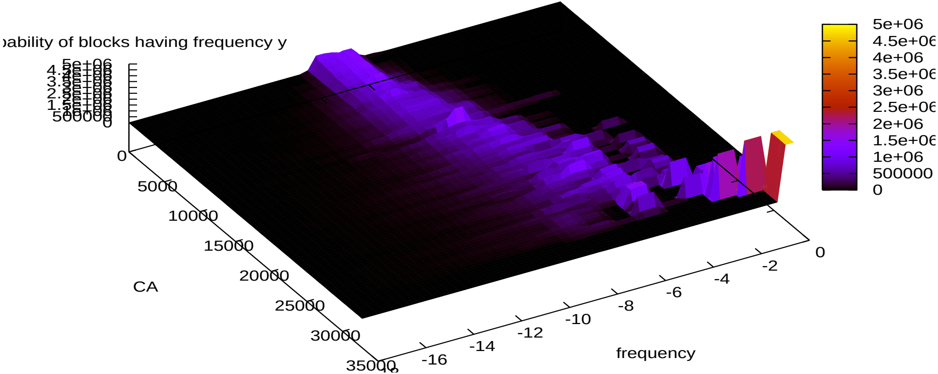
The purple massif next to the red wall is caused by CA whose background pattern has 2 elements (e.g. checkerboard, stripes).

There are 14 with stripes and 8 with checkerboard.

The red ridge is caused by CA which have regular background pattern(s) with 3-16 elements. There are at least 30 of these.

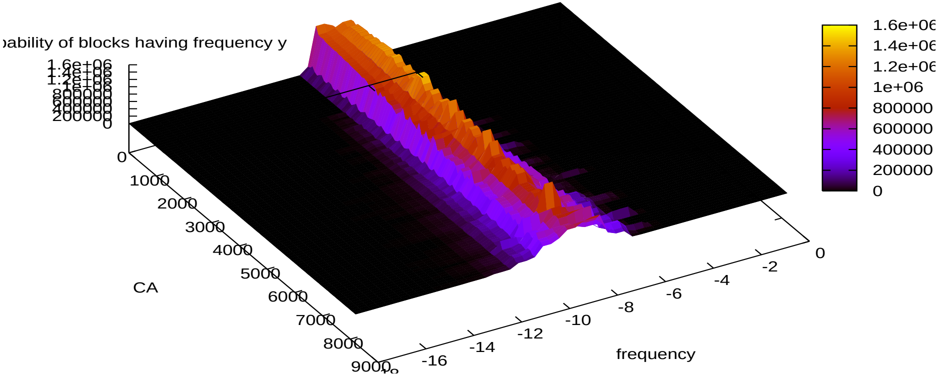
The low blue mounds further left are caused by the low-probability defects.

The probability landscape of Triangle CA



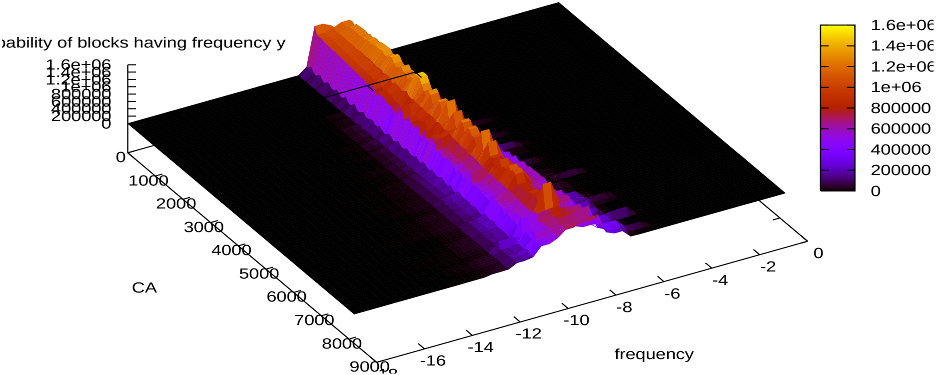
Now, each y value represents one of the 32768 distinct 'triangle' CA.
The picture is very similar to the landscape for 3-cell CA.

Triangle CA landscape; Closeup 0-8191



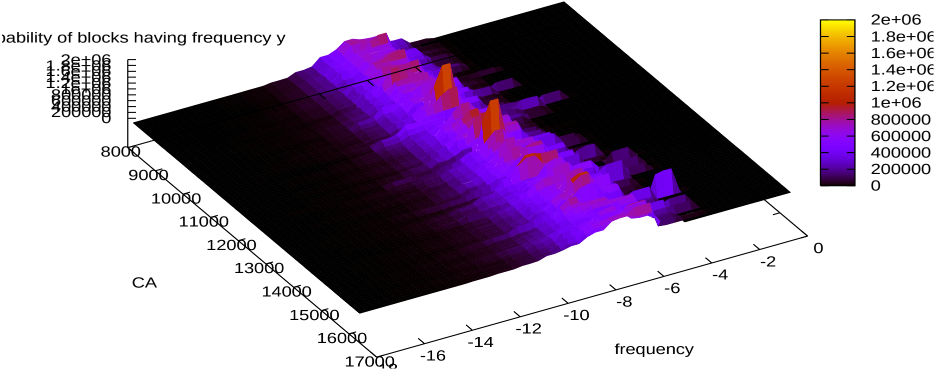
The red ridge is caused by CA which preserve the uniform measure. There are about 3000 of these.

Triangle CA landscape; Closeup 0-8191



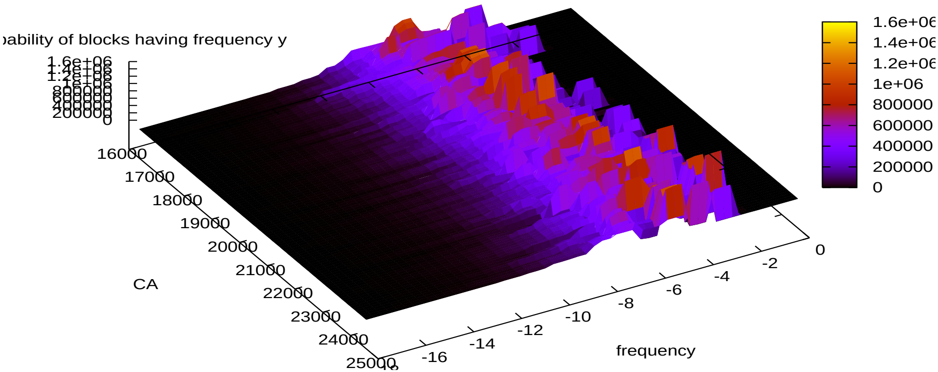
The red ridge is caused by CA which preserve the uniform measure.
There are about 3000 of these.
The red ridge gradually flattens out into CA which 'almost' (but not quite) preserve the uniform measure.

Triangle CA landscape; Closeup 8192-16383



This ‘mountain range’ is caused by CA do not converge quickly to any low-entropy subshift; they exhibit no strong statistical signature of EDD.

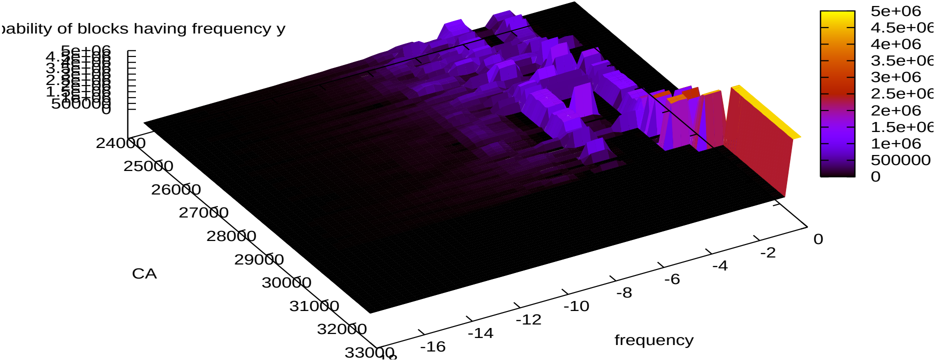
Triangle CA landscape; Closeup 16384-24575



The mountain range continues into the third region.

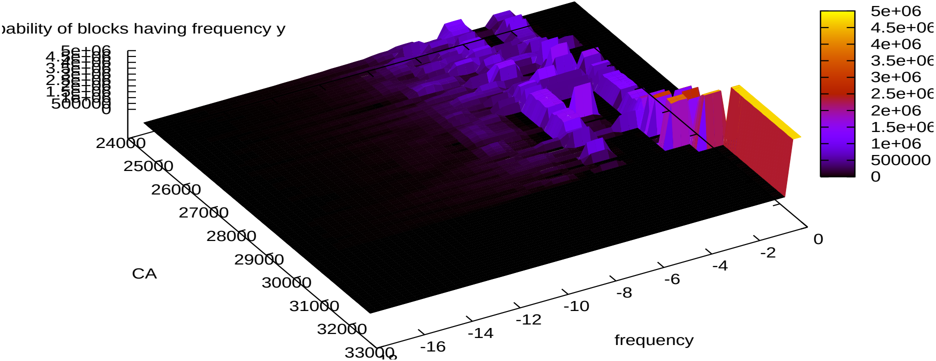
Note: the picture suggests that 3×3 blocks occur with a wide range of frequencies, but this is probably an artifact of small sample size. Each CA was simulated on a 512×512 grid, so there are only $512^2 = 262\,144$ samples per CA, which is insufficient to accurately estimate a probability distribution on the $2^9 = 512$ distinct 3×3 blocks.

Triangle CA landscape; Closeup 24576-32768



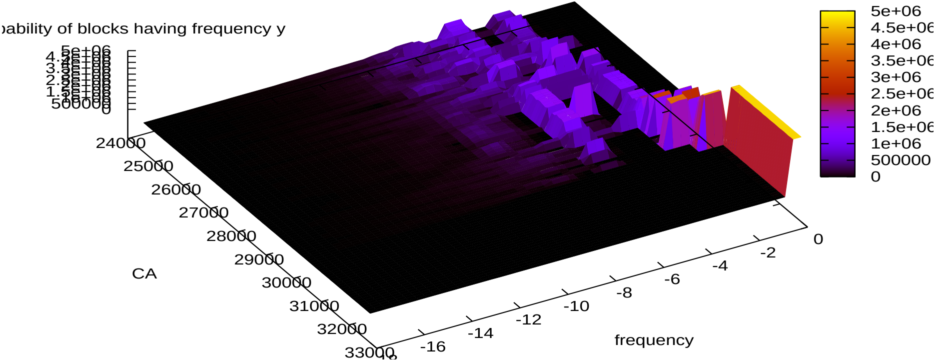
The red ‘wall’ in the right-hand corner is caused by nilpotent CA. There are around 3700 of these.

Triangle CA landscape; Closeup 24576-32768



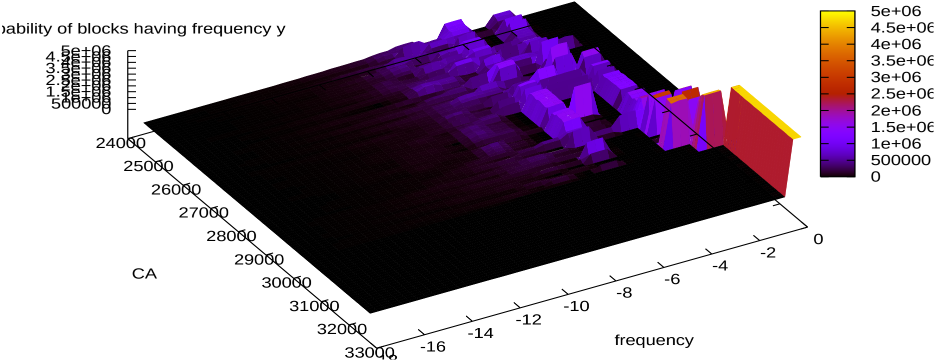
The red 'wall' in the right-hand corner is caused by nilpotent CA.
There are around 3700 of these.
The red and purple 'teeth' near the red wall are caused by CA with EDD.
The far right row of (red) teeth is caused by CA whose background pattern has cardinality 2. (1126 with stripes and 563 with checkerboard).

Triangle CA landscape; Closeup 24576-32768



The red and purple ‘teeth’ near the red wall are caused by CA with EDD. The far right row of (red) teeth is caused by CA whose background pattern has cardinality 2. (1126 with stripes and 563 with checkerboard). The next row of (purple) teeth are cause by CA whose background pattern has cardinality 3-8. (There are around 300 of these).

Triangle CA landscape; Closeup 24576-32768



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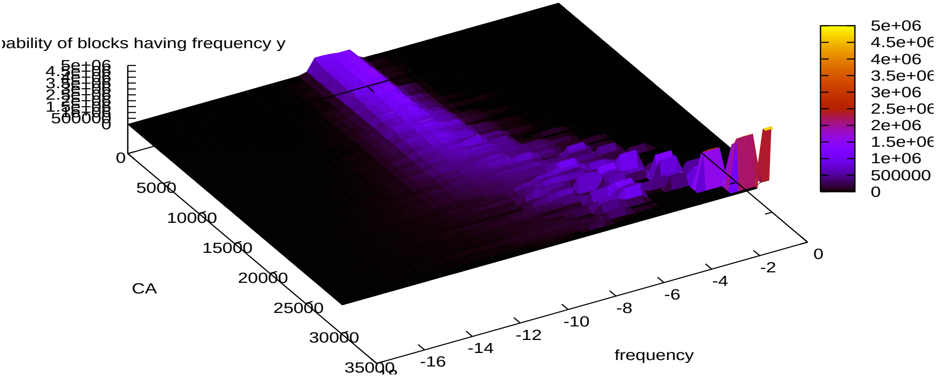
The low blue mounds further left are caused by the low-probability defects.

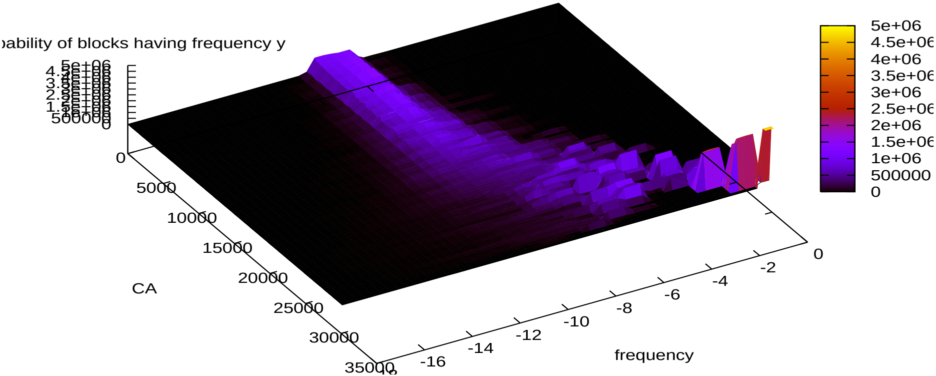
The probability landscape of von Neumann CA

[Skip]



There are 4 294 967 296 distinct VN CA local rules.

The probability landscape of von Neumann CA [Skip]



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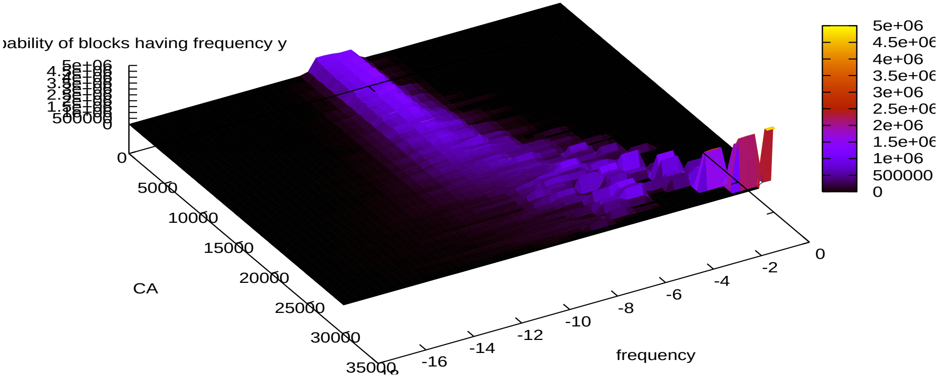
This graph was obtained by randomly sampling 30000 of them.

The probability landscape of von Neumann CA [Skip]



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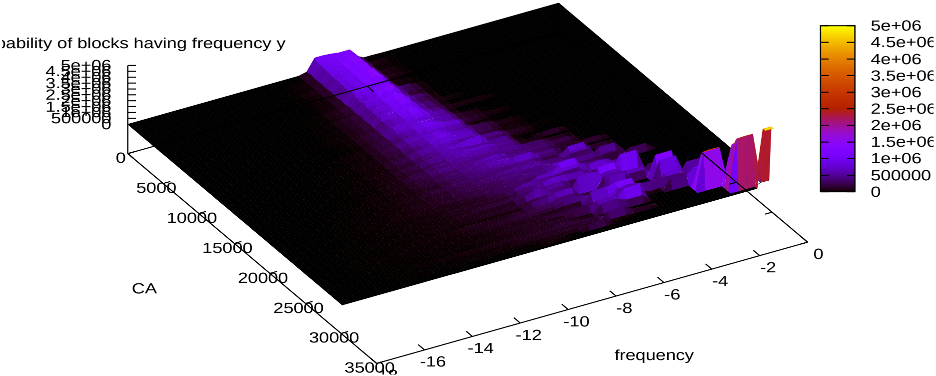
Now, each y value represents one of these 30000 random VN CA.

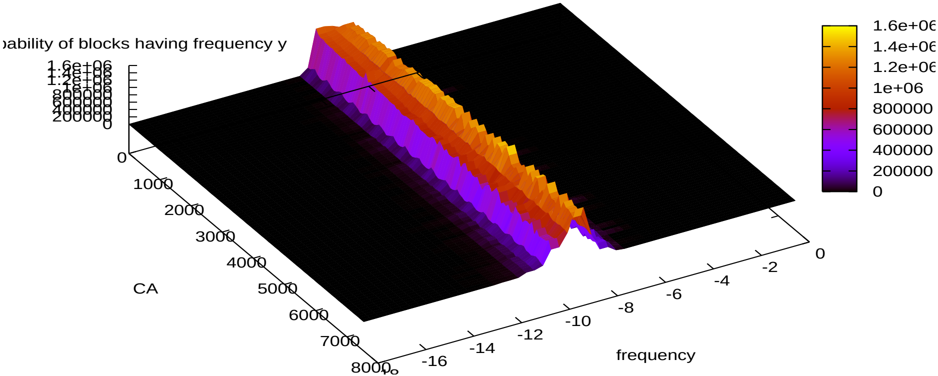
The probability landscape of von Neumann CA

[Skip]



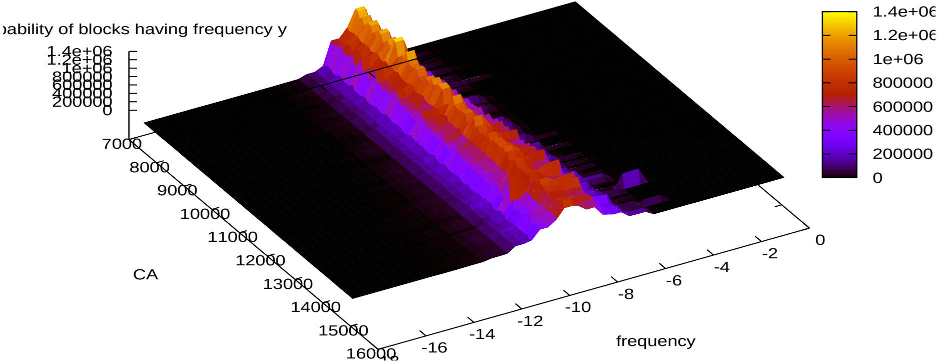
There are 4 294 967 296 distinct VN CA local rules.
This graph was obtained by randomly sampling 30000 of them.
Now, each y value represents one of these 30000 random VN CA.
The picture is similar to the landscape for 3-cell and triangle CA, but the proportion of CA with EDD is much smaller.

von Neumann CA landscape; Closeup 0-7500



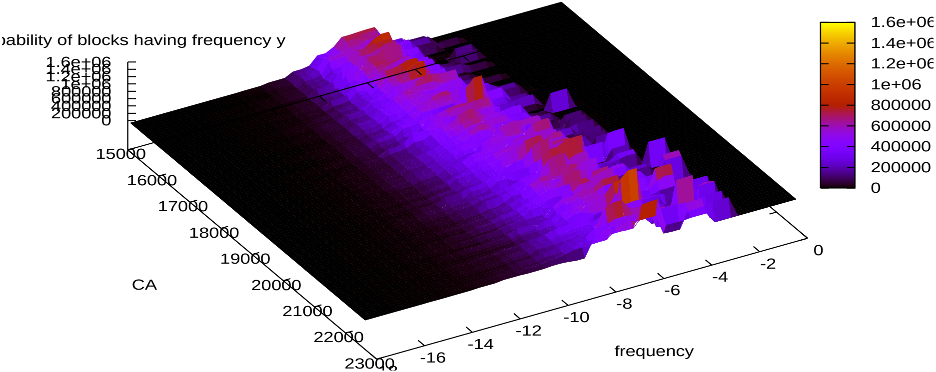
The red ridge is caused by VN CA which preserve uniform measure.

von Neumann CA landscape; Closeup 7500-15000



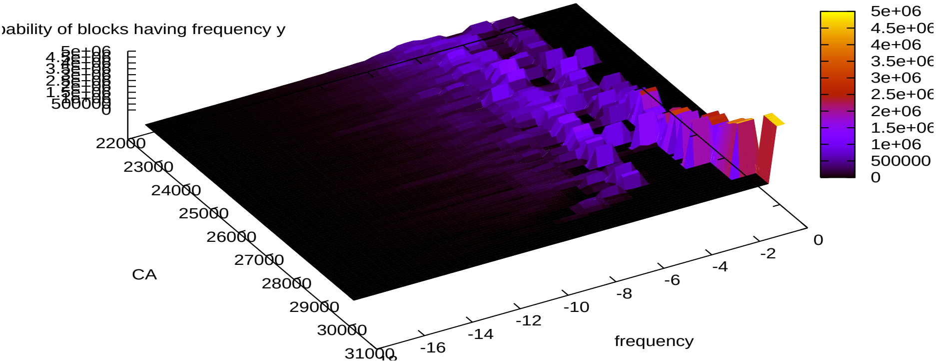
The red ridge (caused by CA which almost preserve uniform measure) continues into this frame.

von Neumann CA landscape; Closeup 15000-22500



This ‘mountain range’ is caused by CA which don’t rapidly converge to any low-entropy subshift; they exhibit no strong statistical signature of EDD.

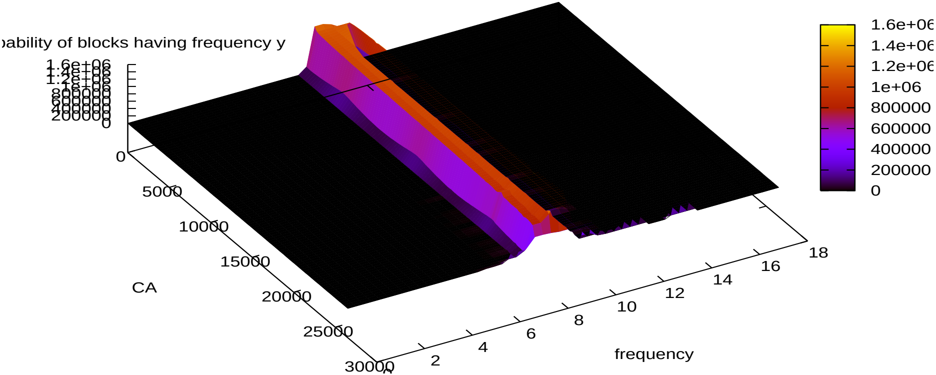
von Neumann CA landscape; Closeup 22500-30000



The red spike in the right-hand corner is caused by nilpotent CA.

The purple teeth next to the red spike are caused by CA exhibiting EDD.

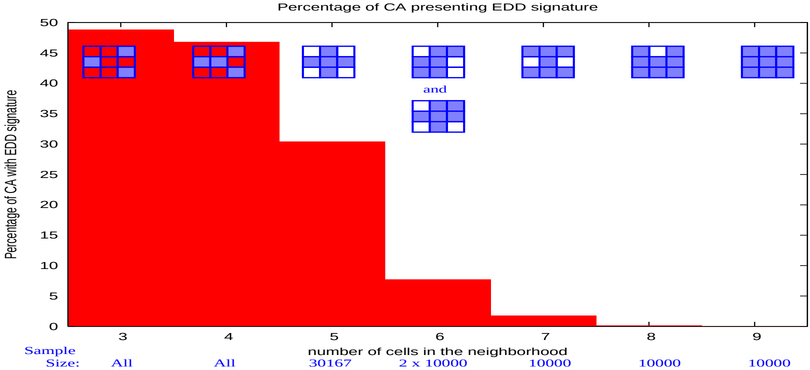
The probability landscape of Moore CA



Of the more than 25000 Moore CA we tested, *none* exhibited a strong statistical signature of EDD.

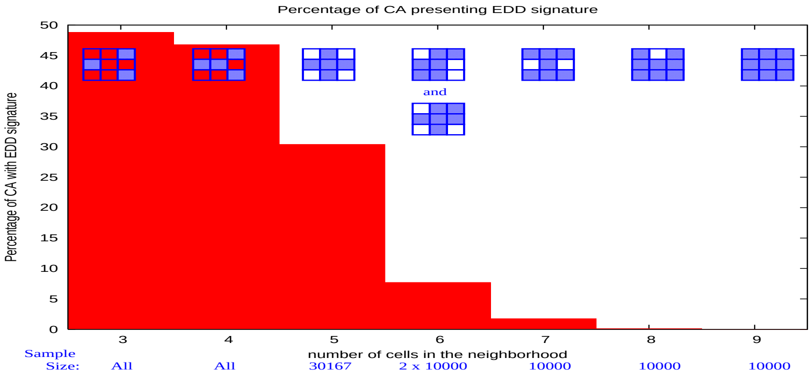
Indeed, it appears that the vast majority ‘almost-preserve’ the uniform measure.

EDD vs. Neighbourhood size



One puzzling phenomenon is that the proportion of CA exhibiting EDD declines very sharply as the neighbourhood size increases.

EDD vs. Neighbourhood size



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Almost 50% of CA with the 3-cell neighbourhood CA exhibit EDD, as do a comparable proportion of CA with the Triangle (4 cell) neighbourhood.

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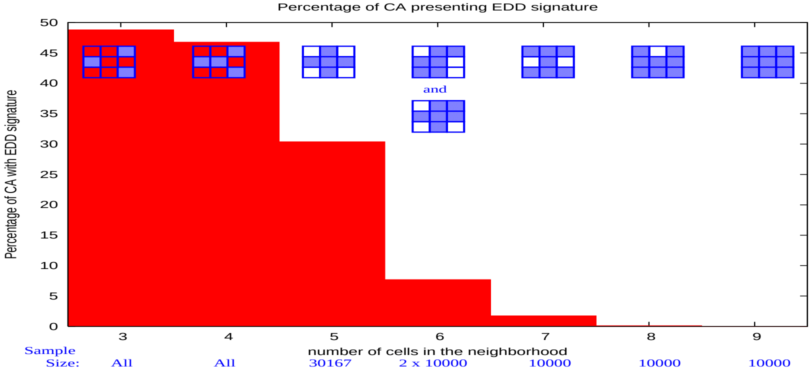

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However, only 30% of CA with von Neumann (5 cell) neighbourhood show a statistical signature for EDD.

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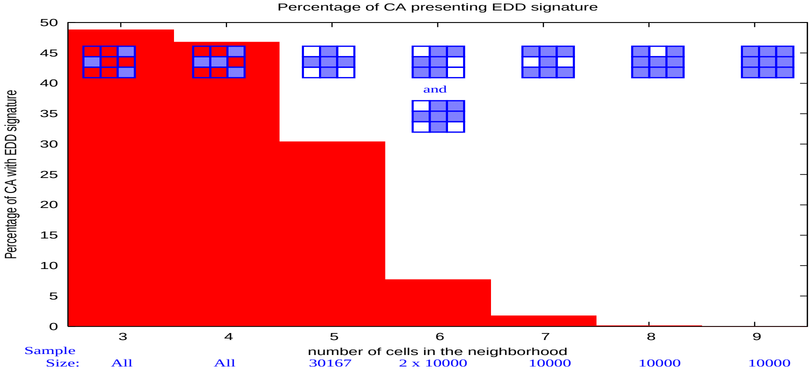

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CA with larger neighbourhoods have even less. The proportion of EDD in CA with the Moore neighbourhood (9 cells) is virtually zero.

How to spot domain boundaries and other defects

Let Φ be a CA, let $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^2}$, and let $\mathbf{b} := \Phi^{100}(\mathbf{a})$.

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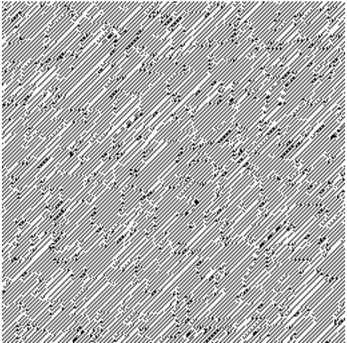
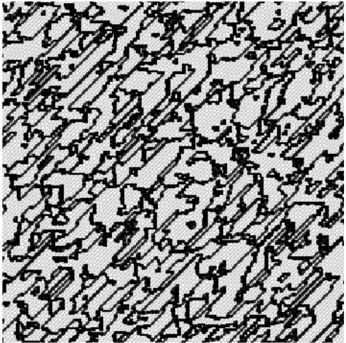
CA△6284

Raw image ($\mathbf{b}$)



Filtered image ($\mathbf{g}$)



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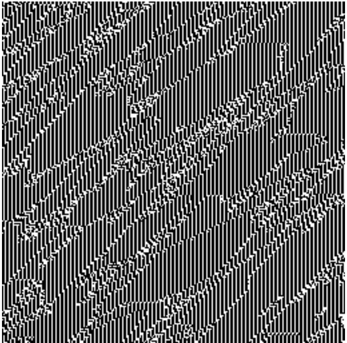
CA△6042

Raw image ($\mathbf{b}$)



Filtered image ($\mathbf{g}$)

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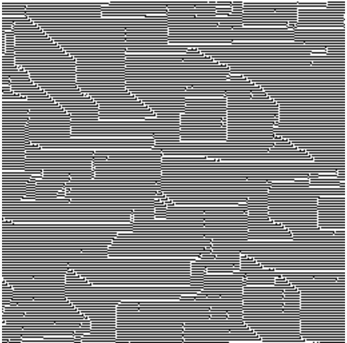
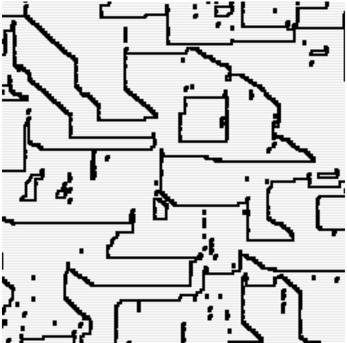
CA△4714

Raw image ($\mathbf{b}$)



Filtered image ($\mathbf{g}$)



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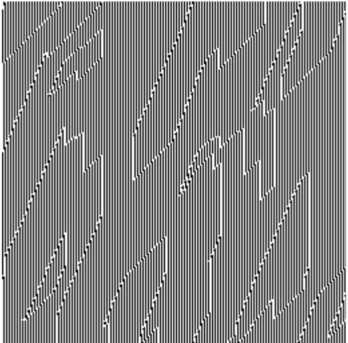
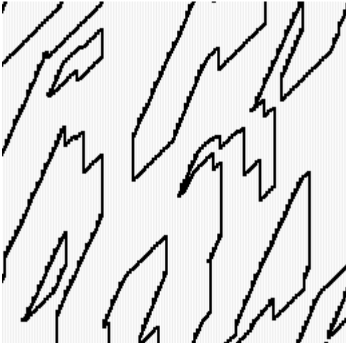
CA△61

Raw image ($\mathbf{b}$)



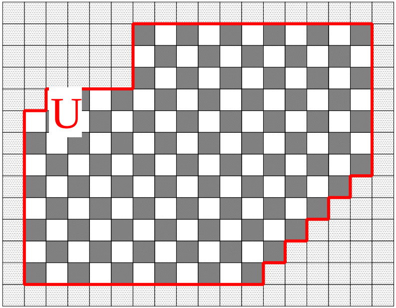
Filtered image ($\mathbf{g}$)



How to identify periodic patterns

Let Φ be a CA. Let $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^2}$ be an initial configuration, and let $\mathbf{b} := \Phi^{100}(\mathbf{a})$.

How to identify periodic patterns



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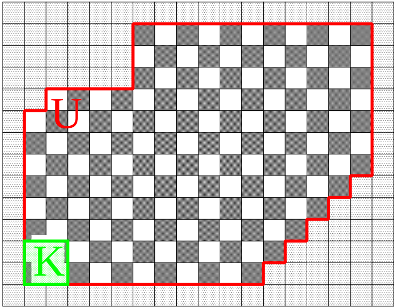
How to identify periodic patterns



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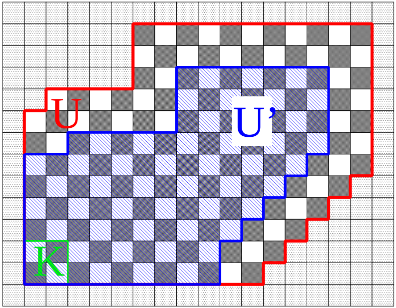

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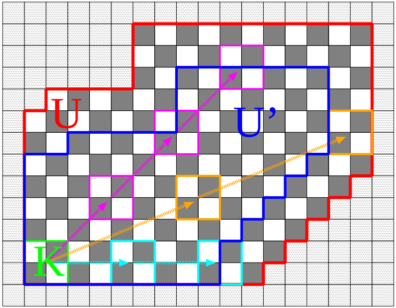
Let $\mathbb{K} := [0...k)^2 \subset \mathbb{Z}^2$. Let $\mathbb{U}' := \mathbb{U} \setminus (\mathbb{U}^{\complement} - \mathbb{K})$ be the '$\mathbb{K}$-interior' of $\mathbb{U}$.

How to identify periodic patterns



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Let $\mathbb{S} := \{\mathsf{u} - \mathsf{v}\ ;\ \mathsf{u}, \mathsf{v} \in \mathbb{U}', \mathbf{b}_{\mathsf{u}+\mathbb{K}} = \mathbf{b}_{\mathsf{v}+\mathbb{K}}\}$.

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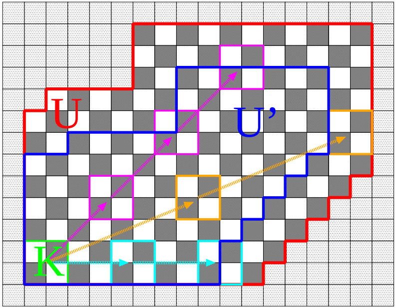

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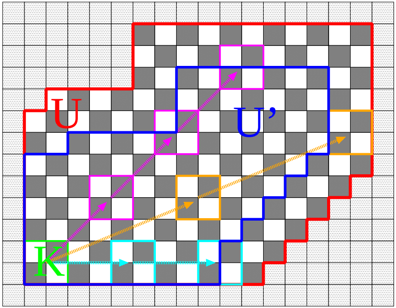
Let $\mathbb{S} := \{\mathsf{u} - \mathsf{v} \; ; \; \mathsf{u}, \mathsf{v} \in \mathbb{U}', \mathbf{b}_{\mathsf{u}+\mathbb{K}} = \mathbf{b}_{\mathsf{v}+\mathbb{K}}\}$. If $\mathbb{P} \subset \mathbb{Z}^2$ is subgroup, then

$$\Big(\mathbb{U} \text{ has } \mathbb{P}\text{-periodic pattern}\Big) \iff \Big(\mathbb{S} = (\mathbb{U}' - \mathbb{U}') \cap \mathbb{P}\Big).$$

How to identify periodic patterns



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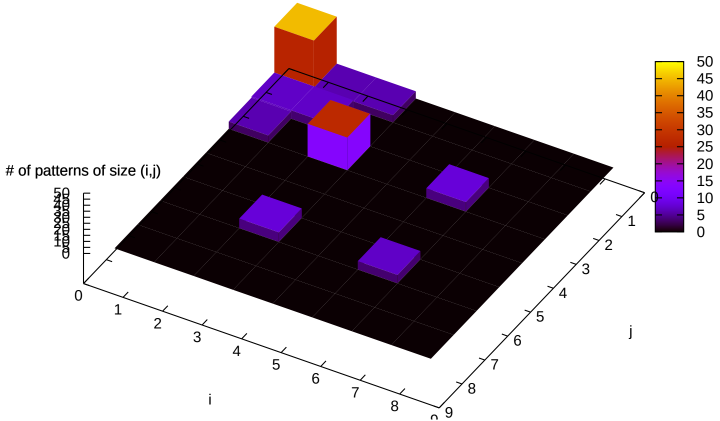
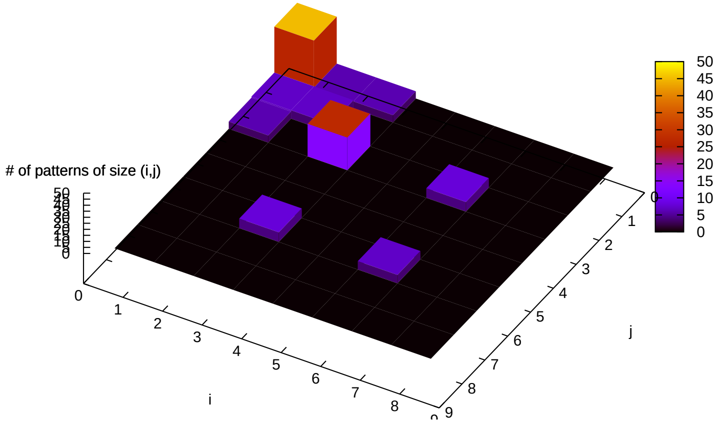
(The search for this *periodicity group* $\mathbb{P}$ can be automated.)

The distribution of periodic structures: 3-Cell CA



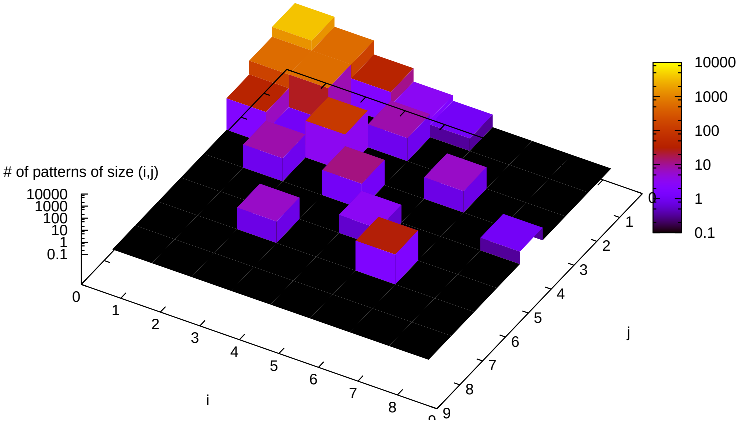
The empirical frequency of periodic patterns in 3-cell CA.

The distribution of periodic structures: 3-Cell CA



The empirical frequency of periodic patterns in 3-cell CA. The height of box (i, j) is the number of 3-cell CA whose EDD has an (i, j)-periodic regular domain.

The distribution of periodic structures: Triangle CA



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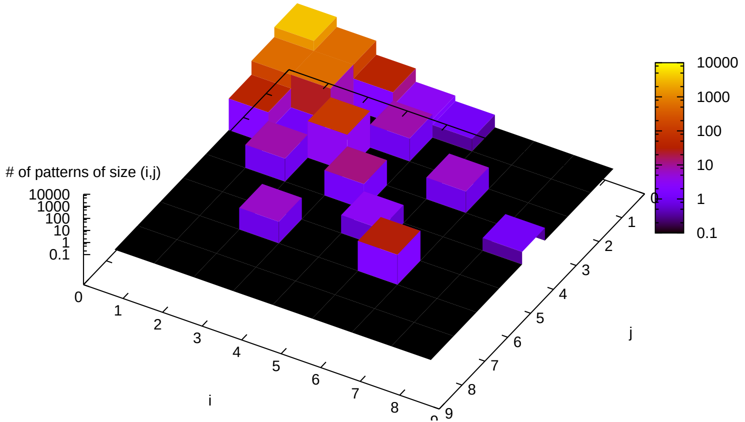

The empirical frequency of periodic patterns in triangle CA. The height of box (i, j) is the logarithm of the number of $\triangle$ CA whose EDD has an (i, j)-periodic regular domain.

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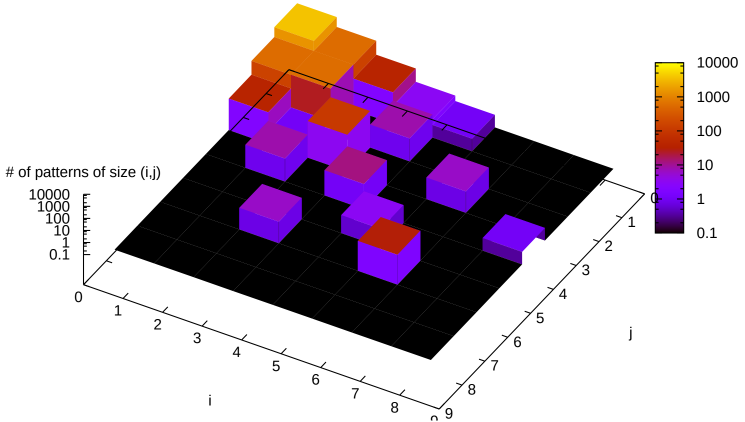

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Note: graph is not symmetric; e.g. height$(0,3)$ $\neq$ height$(3,0)$.

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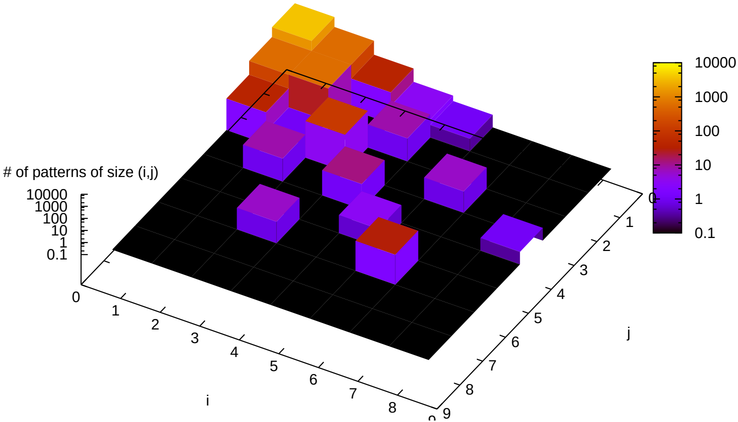

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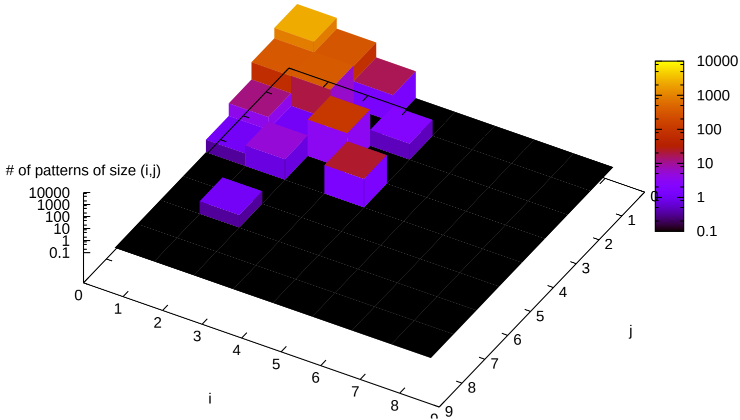
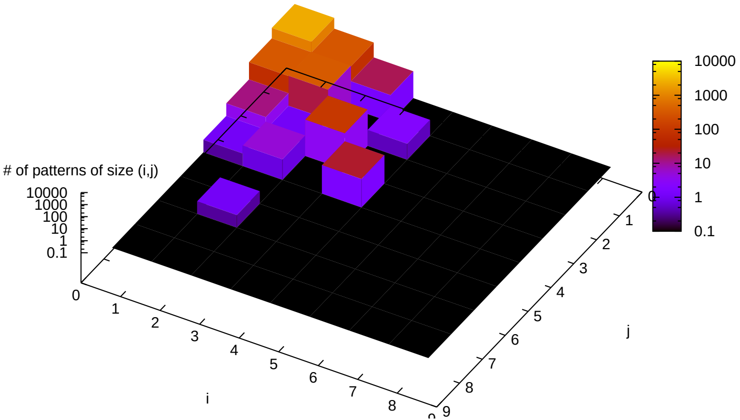
The distribution of periodic structures: von Neumann CA



The empirical frequency of periodic patterns in von Neumann CA.

The distribution of periodic structures: von Neumann CA



The empirical frequency of periodic patterns in von Neumann CA. The height of box (i, j) is the log of the number of vN CA whose EDD has an (i, j)-periodic regular domain.

How to spot interfaces and dislocations

Let Φ be a CA. Let $\mathbf{a} \in \mathcal{A}^{\mathbb{Z}^2}$ and $\mathbf{b} := \Phi^{100}(\mathbf{a})$. Let $\mathbb{K} := [0...k)^2 \subset \mathbb{Z}^2$.

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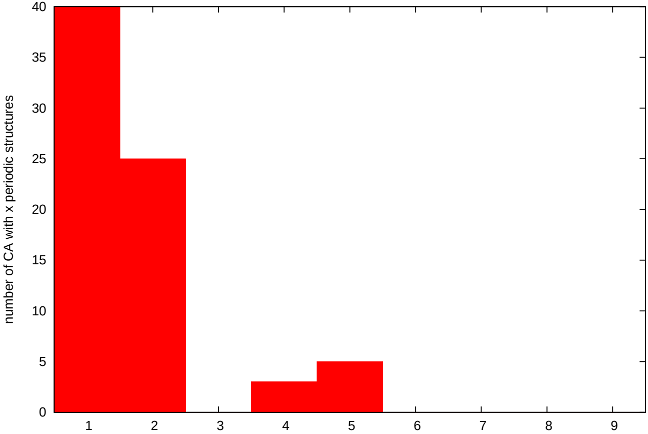
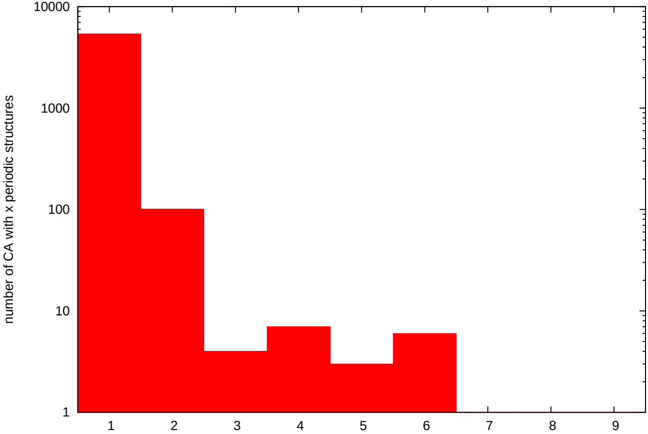
We can visualize this by colouring each domain according to its phase.
By thus colourizing an entire spacetime animation, we can see domain growth and defect motion over time.

The frequency of interfaces: 3-Cell CA



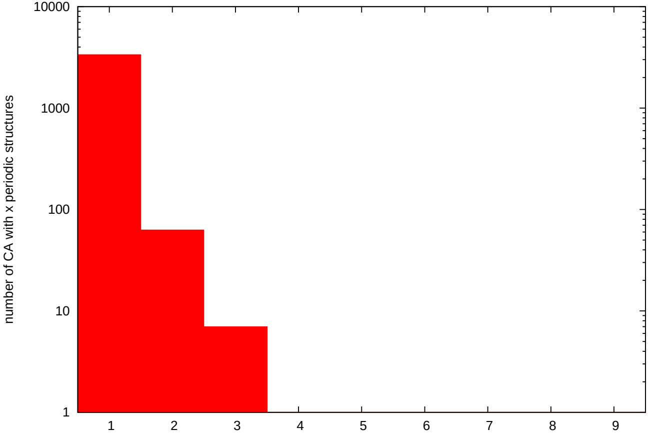
For each $n \in \mathbb{N}$, this graph shows the number of the 256 distinct 3-Cell CA whose emergent defect dynamics exhibits at least n distinct periodic subshifts (and hence, $n(n-1)/2$ possible interface types).

The frequency of interfaces: Triangle CA



For each $n \in \mathbb{N}$, this graph shows the log-number of the 32768 distinct Triangle CA whose emergent defect dynamics exhibits at least n distinct periodic subshifts (and hence, $n(n-1)/2$ possible interface types).

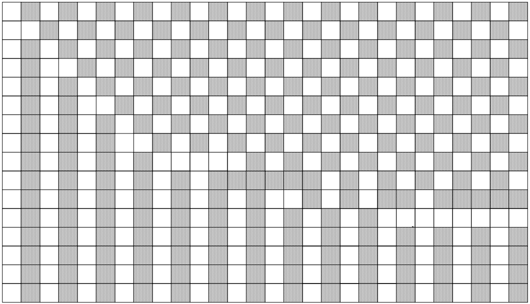
The frequency of interfaces: vN CA



For each $n \in \mathbb{N}$, this graph shows the log-number out of a random sample of 3276 vN CA whose emergent defect dynamics exhibits at least n distinct periodic subshifts (and hence, $n(n-1)/2$ possible interface types).

Boundary dynamics

[Skip]

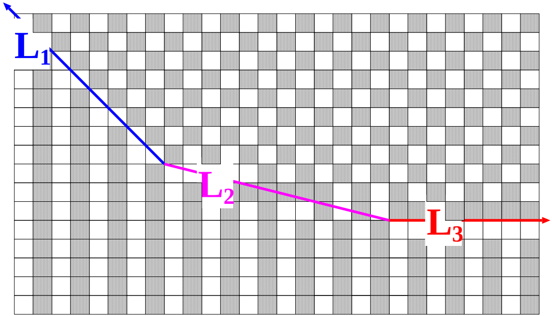


Consider a boundary between two regular domains.

How can we mathematically model the motion of this boundary?

Boundary dynamics

[Skip]



$r_1 = -1/2.$

$r_2 = -1/4.$

$r_3 = 0.$

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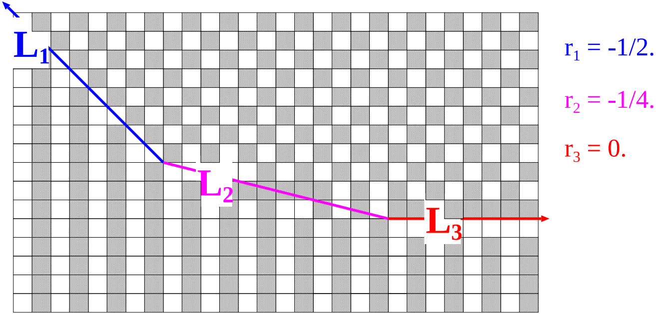
1. Construct polygon along boundary, using line segments $\mathbf{L}_1, \mathbf{L}_2, \ldots, \mathbf{L}_N$ with rational slopes $r_1, \ldots, r_N$.

Boundary dynamics

[Skip]



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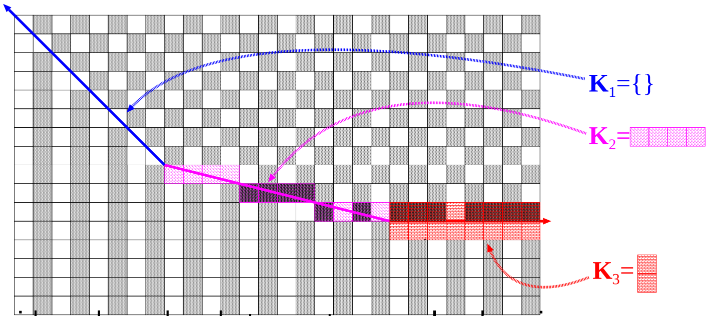

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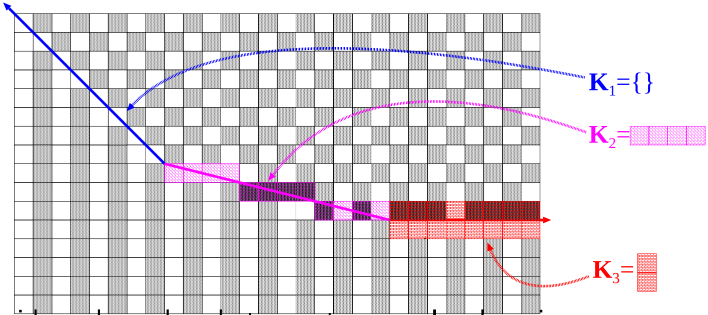
2. For all n, let $\mathbb{K}_n \subset \mathbb{Z}^2$ be a minimal subset such that the defect along $\mathbf{L}_n$ can be tiled with ℓ_n disjoint translates of $\mathbb{K}_n$ along $\mathbf{L}_n$.

Boundary dynamics

[Skip]



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Boundary dynamics

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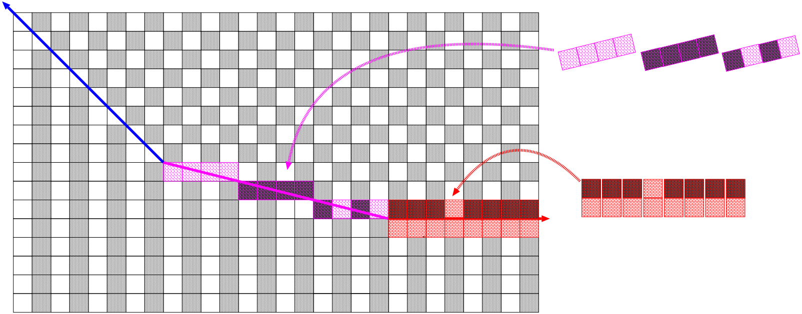

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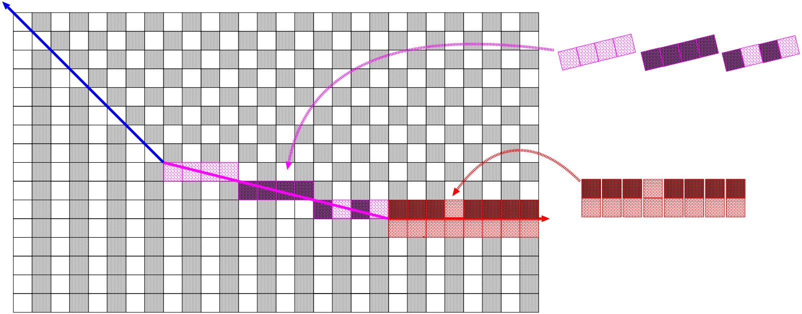

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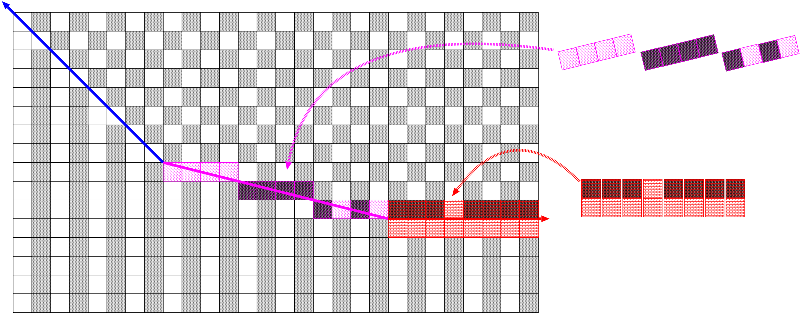

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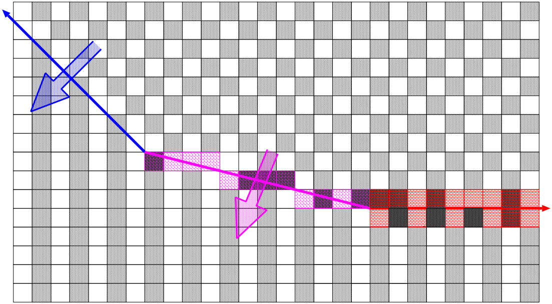

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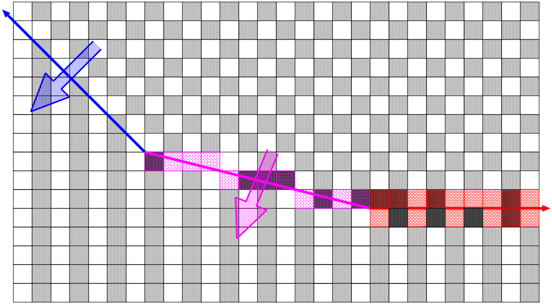

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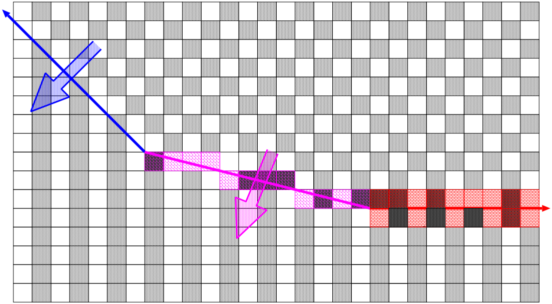

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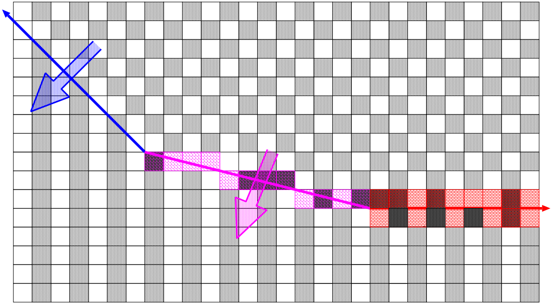

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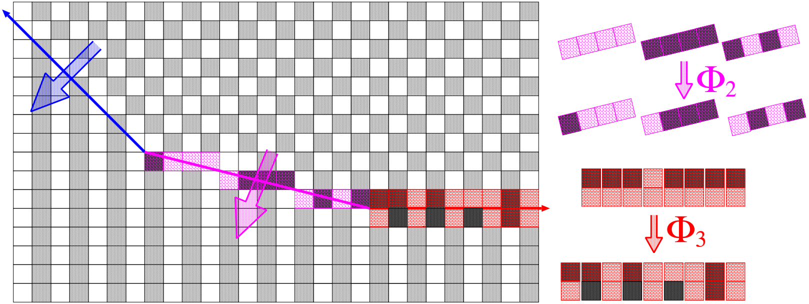

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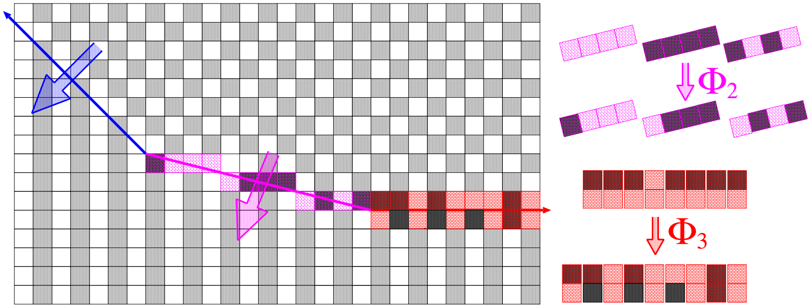

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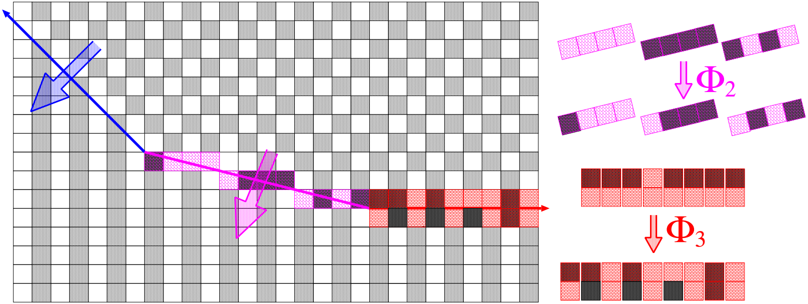

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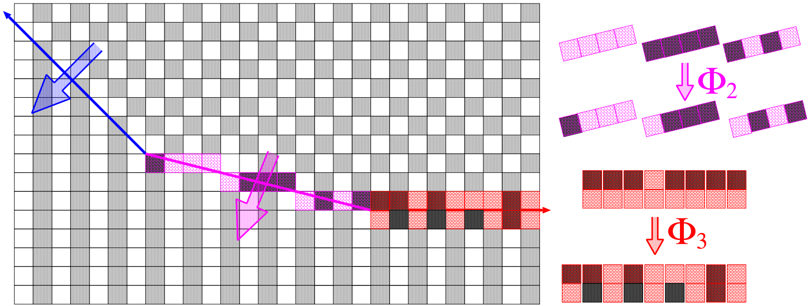

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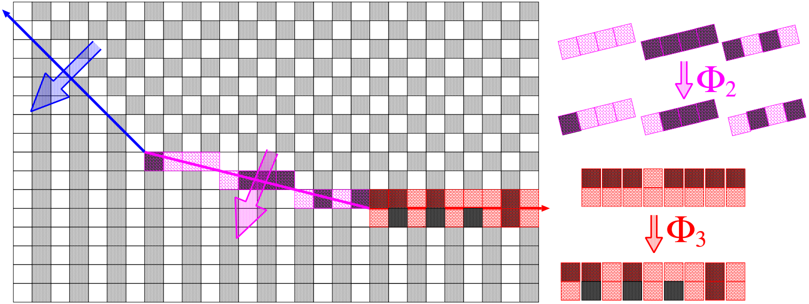

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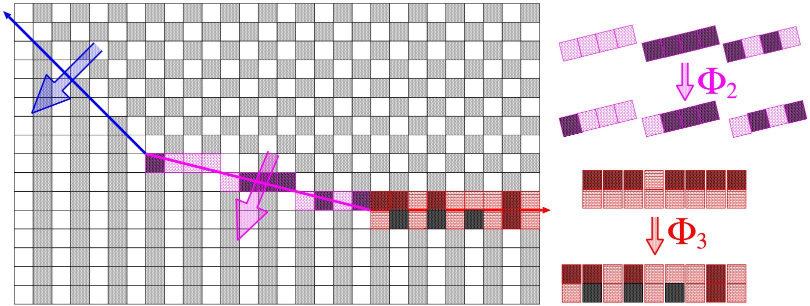

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  However, we have no algorithm to automatically detect 'gaps' or 'poles' (this requires the automatic detection of a height function). Thus we have no idea of their frequency.

Literature

- *Defect Particle Kinematics in One-Dimensional Cellular Automata*, M. Pivato, *Theoretical Computer Science*, **377**, (#1-3), May 2007, pp.205-228. `http://arxiv.org/abs/ math.DS/0506417`
- *Algebraic Invariants for Crystallographic Defects in Cellular Automata*, M. Pivato, *Ergodic Theory & Dynamical Systems*, **27** (#1), February 2007, pp. 199-240. `http://arxiv.org/abs/math.DS/0507167`
- *Spectral Domain Boundaries in Cellular Automata*, *Fundamenta Informaticae*, **78** (#3), 2007, pp.417-447. `http://arxiv.org/abs/math.DS/0507091`

Please go to `http:euclid.trentu.ca/Defect` to obtain:

- The complete slides for this talk.
- The raw data on EDD in two-dimensional CA.
- The source code for the software we used to obtain this data.

Introduction

ECA #62, 184, 110, 18, etc.
Past empirical/theoretical work

Algebraic invariants

Interfaces
Dislocations
Gaps
Poles