Mathematics 3830H – A Survey of the History of Mathematics

TRENT UNIVERSITY, Winter 2025

TAKE-HOME FINAL EXAMINATION Due on Thursday, 17 April.*

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may ask the instructor to clarify the instructions or any of the questions, use a calculator or computer to perform any necessary calculations, and consult any sources you wish, with the exception of other students' work, past or present, and you may not give or receive any other aid on this exam, except with the instructor's explicit permission.

Part I – This and that, ... Do all three of 1 - 3.

- **1.** Answer all of $\mathbf{a} \mathbf{i}$. $(10 = 10 \times 1 \text{ each})$
 - **a.** Which mathematical tradition used the method of false position?
 - **b.** Name three mathematicians who were also civil servants, in positions unrelated to their careers as mathematicians.
 - c. Name three mathematicians who were also inventors or engineers.
 - **d.** Who first used + to denote addition?
 - e. Who first systematically used a symbolic notation for algebraic expressions?
 - f. Why were reciprocals so important in Mesopotamian mathematics?
 - **g.** Which of the number systems mentioned in this course required the fewest distinct symbols and which required the most?
 - **h.** Who seems to have been the first to do proofs in mathematics?
 - i. Name three mathematicians who were also astrologers.
 - **j.** What were complex numbers invented for?
- 2. What features would need to be added the Mesopotamian numbers system to make it comparable in capability to the modern Hindu-Arabic number system? Given that these were added, what would the remaining advantages and disadvantages of these number systems be? [10]
- 3. Recall Ptolemy's Theorem from class, slightly rephrased here:

Suppose A, B, C, and D are any four points on a circle listed in clockwise order. Then $|AB| \cdot |CD| + |AD| \cdot |BC| = |AC| \cdot |BD|$.

Recall also that the chord function gives the length of a chord of a circle subtended by a given central angle. Use Ptolemy's Theorem to prove the following identity for the chord function (in a unit circle, with angles in radians):

 $\operatorname{chord}(\alpha - \beta) = \operatorname{chord}(\alpha)\operatorname{chord}(\pi - \beta) - \operatorname{chord}(\beta)\operatorname{chord}(\pi - \alpha) \quad [10]$

Hint: Make AD a diameter in Ptolemy's Theorem and let α and β be the angles ...

[Parts II – IIII are on page 2.]

^{*} Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca.

Part II – ... and has-beens, ... Do any two (2) of 4 - 6.

- 4. Read The architect Kha's protractor by Amelia Carolina Sparavigna, which you can find at https://arxiv.org/abs/1107.4946, plus whatever additional sources you can find about the artifact in question. Summarize the theories about what the artefact is and assess the evidence for and against each theory. [10]
- 5. Find out about and describe the so-called Antikythera Mechanism. What does it tell us about the capabilities of Classical Greek and Hellenistic engineering, astronomy, and mathematics? [10]
- 6. Was the development of a place-value system for writing numbers necessary for the development of modern algebra and calculus? Justify your assertion! [10]

Part III – ... and necessary things, ... Do any two (2) of 7 - 10.

- 7. Show that *Gregory's series*, $4 \frac{4}{3} + \frac{4}{5} \frac{4}{7} + \frac{4}{9} \cdots$, converges to π and determine how many terms of it one must one add up to guarantee that the partial sum is within 0.01 of π . [10]
- 8. Problem 40 of the Rhind Papyrus, translated into modern English and very slightly modified, is:

Divide one hundred loaves among five persons in such a way that the shares shall be in arithmetic progression and that one-seventh of the sum of the largest three shares shall be equal to the sum of the smallest two. What are the shares?

Use the method of false position to solve this problem, showing all the steps. [10]

- **9.** A natural number n is congruous if there are natural numbers a and b with a > b such that n = ab(a b)(a + b) (if a + b is even) or n = 4ab(a b)(a + b) (if a + b is odd). Prove Fibonacci's result that if n is congruous, then it is divisible by 24. [10]
- 10. Show that any ellipse can be generated by some epicycle. [10]

|Total = 70|

Part IIII – ... and even verse things. Bonus!

- $\iota \alpha$. Write an original poem touching on mathematics or its history. [1]
- $\iota\beta$. Find and give a complete reference to a poem touching on mathematics or its history that you did not write, and which your instructor has not seen before. [1]

I HOPE THAT YOU ENJOYED THE COURSE. ENJOY THE SUMMER!