# Diophantus of Alexandria (c. 250 A.D. ± 100 or so...)

- we know almost nothing about him, witness

- his major work is the <u>Arithmetica</u>
  - originally had 13 books
  - 6 have survived
  - 4 more may have survived in Arabic translation (tentatively identified in 1973, but not everyone agrees they are part of the Arithmetica)
  - 3 are completely lost
  - it was lost or ignored from his death to the 10th century A.D. until it was translated into Arabic by Abu al-Wafa Buzjani (940-988), but became quite influential after this

The <u>Arithmetica</u> is principally concerned with numerical solutions to algebraic equations with positive coefficients. In some ways it is a throwback to Mesopotamian & Egyptian mathematics. There are very few proofs or references to such, almost all of it is solutions to particular problems, though the methods used generalize.

One major innovation was a shorthand for representing equations that is the first cut at a symbolic algebraic notation.

| Operation/expression | denoted by |
|---|---|
| + | juxtaposition |
| − | $\pitchfork$ |
| = | ἴσ ~~[illegible]~~ |
| constant | $\mathring{M}$ |
| unknown | $\varsigma$ ($\varsigma\varsigma$ if coeff. > 1) |
| unknown squared | $\Delta^{\upsilon}$ |
| unknown cubed | $K^{\upsilon}$ |
| integers coefficients | Ionic system — " ——— with bars on top, written after the unknown. |

Examples:

1) $\Delta^{\upsilon}\bar{\gamma}\,\mathring{M}\overline{\iota\beta}$

we'd write as $3x^2+12$

2) $K^{\upsilon}\bar{\alpha}\,\varsigma\varsigma\bar{\eta} \pitchfork \Delta^{\upsilon}\bar{\epsilon}\,\mathring{M}\bar{\alpha}$ ἴσ $\mathring{M}\bar{\alpha}$

represents what we'd write as

$(x^3+8x)-(5x^2+1)=1$

This system did have severe limitations:

- the underlying number system could handle fractions, but had no zero or negatives
- when dealing with two unknowns, one had to fall back on expressing most things in words
- in particular, there was no symbol for division, so rational functions could only be handled in words

... but it was still far more compact and convenient than expressing everything in words.

Here are a couple of problems (8 and 9) from Book II of the Arithmetica, to give you a flavour of it:

Problem 8: To divide a square into a sum of two squares.

Solution: (Using our notation for his shorthand.) To divide 16 into a sum of two squares. Let the first summand be $x^2$, and thus the second $16-x^2$. The latter is to be a square. I form the square of the difference of an arbitrary multiple of x diminished by the root of 16, that is, diminished by 4. I form, for example, the square of $2x-4$. It is $4x^2+16-16x$. I put this expression equal to $16-x^2$. I add to both sides $x^2+16x$ and subtract 16. In this way I obtain $5x^2=16x$. Thus one number is $\frac{256}{25}$ and the other $\frac{144}{25}$. The sum is 16 and each summand is a square.

[Fermat scribbled his statement of Fermat's Last Theorem in the margin of his copy of the Arithmetica, but wrote that the margin was too small to write the proof down.]

[Guess which problem he wrote next to...]

Problem 9: To divide a given number which is the sum of two squares into two other squares.

Solution: Suppose 4 is the first given square and 9 is the second given square. Take $(x+2)^2$ as the first square and $(mx-3)^2$ as the second square, where m is an integer, say $m=2$.

Therefore $(x^2+4x+4)+(4x^2+9-12x)=13$, or

$$5x^2+13-8x=13.$$

Therefore $x=\frac{8}{5}$ and the required squares are $\frac{324}{25}$ and $\frac{1}{25}$.

[For a general solution, let m be any rational.]

Except in the few places that Diophantus gives or refers to general results, it's hard to be sure just how much he knew or could prove. For example, in Problem 19 of Book III, Diophantus notes that 65 is the sum of two squares in two different ways "since 65 is the product of 13 and 5, each of which is the sum of two squares." One could deduce he knew of the general result here, but the first one to actually write down the relevant fact, $(a^2+b^2)(c^2+d^2)=(ac\mp bd)^2+(ad\pm bc)^2$, was another 10th Century Muslim mathematician, Abu Jafar al-Khazin. People still argue whether Diophantus really knew this or not.