Joseph-Louis Lagrange (1736-1813)

Born in Turin, Italy, with mixed Italian & French ancestry, as Giuseppe Lodovico Lagrangia. Attended the University of Turin, initially to study law, but switched to math. He was appointed an assistant professor at the university in 1755. Moved on to the Academy of Sciences in Berlin after Euler left in 1766, and then on to Paris in 1786. [Just a little before the Revolution.] His sponsor in Paris, the nobleman & chemist Antoine Lavoisier (1743-1794) was executed during the terror. Lagrange made plans to escape, but ended being honoured with every change of regime and died of natural in Paris in 1813.

Worked in a number of areas of physics & mathematics, plus the occasional bit of consulting - he (and Lavoisier before him) had a significant part of developing the metric system.

In physics, he worked on mechanics and astronomy, acoustics, etc. (The Lagrange "points" of an orbiting body where a small mass may stay in a stable relative position were identified by him.) He also simplified Newtonian mechanics using the notion of potential energy, and provided an explanation for why the Moon is always presenting one face to the Earth. (Tidal locking ...)

Along the way he developed a lot of applied mathematics, eg Lagrange multipliers for finding extrema in multiple variables. He pioneered the study of partial differential equations in a systematic way, and made major contributions to the calculus of variations.

Made major contributions to algebra, anticipating a good part of Galois & Abel's invention of group theory.

One of the basic results in group theory is the fact that the order of a subgroup divides the order of the group, (# elements) and this was first observed by Lagrange.

Major contributions to Diophantine equations, in particular giving complete solutions to equations of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

where A to F are given integers.

One of his major results was showing that every positive integer is the sum of at most four squares.

<u>Sketch:</u> 1° Every odd prime of the form $4n+1$ is the sum of two integer squares. (Also, $2 = 1^2+1^2$, ...)

2° Every odd prime of the form $4n+3$ is the sum of at most four squares.

3° Show that if m & n are sums of at most four squares then so is $m \cdot n$.

For the last part, Lagrange gave the following formulas:

$$(a^2+b^2+c^2+d^2)(r^2+s^2+t^2+u^2)$$

$$= (ar+bs+ct+du)^2 + (as-br+cu-dt)^2$$
$$+ (at-bu-cr+ds)^2 + (au+bt-cs-dr)^2$$

[Expand & cancel & collect on both sides ...]