Two major Indian mathematicians after Aryabhata: Bhaskara (I) c. 600-680 A.D.

& Brahmagupta c. 600-670 A.D.

Bhaskara was probably from Maratha in central India.

Also a mathematical astronomer, in particular, he wrote a commentary on Aryabhata's work, the Aryabhatiyabhasya.

It's the first surviving work on Indian astronomy and math that is in prose instead of poetry.

- He wrote most of the numbers out in words but often repeated them using the Brahmi numerals with ∘ for zero.
- He also dealt with negative numbers, which he interpreted as deficits to be made up.

Also wrote a major work on astronomy of his own, the Mahabhaskariya which was later translated into Arabic.

In this work he used a rational approximation to sin(x), namely

$$\sin(x) \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)} .$$

This gives results within 2% of the correct value for $0 \leq x \leq \frac{\pi}{2}$.
He claims that this approximation is due to Aryabhata.

He also, as a mathematician, worked on:

- Diophantine equations the Pell equation $8x^2 + 1 = y^2$ where he used the basic solution $x = 1$ & $y = 3$ to obtain a larger solution $x = 6$ & $y = 17$.

- algebra
- spherical trigonometry and geometry

Brahmagupta was from Rajasthan in northwest India.

Wrote two major works that were later translated into Arabic;

the *Brahmasphutasiddhanta* (theoretical work c. 630 A.D.)
and *Khandakhadyaka* (textbook on the practical side of astronomy).

He treated zero and negative numbers as numbers in their own right and developed rules for using them. (Ran into problems trying to figure out $\frac{0}{0}$ and dividing by 0 in general.) Treated fractions in a modern manner as well.

Remembered for his work in algebra & geometry.

In algebra we have, in his work, the (equivalent of the) formulas

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

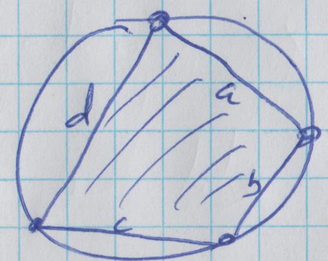
Also worked on Diophantine equations and, in particular, on the Pell equation $x^2 - Ry^2 = 1$ (where $R>0$, $R \in \mathbb{Z}$, and R is not a square).

Used the identity $(x_1^2 - Ry_1^2)(x_2^2 - Ry_2^2) = (x_1x_2 + Ny_1y_2)^2 - N(x_1y_2 + x_2y_1)^2$.

Had pretty general methods for solving linear equations and quadratic equation (these were picked up and improved by al-Kwarizmi).

In geometry, gives formulas for areas & volumes of various shapes (es cones and pyramids), used $\sqrt{10}$ as an approximation to π, and gives various Pythagorean triples (and formulas for generating these).

Best remembered for Brahmagupta's formula (which extends Heron's formula)



$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $s = \dfrac{a+b+c+d}{2}$.

Also generated ~~[illegible]~~ sine tables using interpolation tricks to help comput it. (Use a special case of the Newton-Stirling interpolation formula.

In terms of notation, he didn't have any notation for unknowns but had some for manipulating numbers:

addition was indicated by juxtaposition $a+b$ was "ab"

subtraction ———— " ———— $a-b$ was "$a\dot{b}$"
with a dot over the one being subtracted

division put the divisor below the dividend $\frac{a}{b}$ was $\begin{matrix} a \\ b \end{matrix}$

multiplication (and unknowns) were indicated by abbreviations of words