Therefore second + $\frac{1}{4}$ (third + fourth + first) $= x + 1$,
whence 3 times second + sum of all $= 4x + 4$,
and therefore second $= x + \frac{1}{3}$.
Similarly third $= x + \frac{1}{2}$,
and fourth $= x + \frac{3}{5}$.
Adding, we have $4x + \frac{43}{30} = x + 3$,
and $x = \frac{47}{90}$.

The numbers, after multiplying by a common denominator, are **47, 77, 92, 101**.

26. Given two numbers, to find a third number which, when multiplied into the given numbers respectively, makes one product a square and the other the side of that square.

Given numbers 200, 5; required number x.
Therefore $200x = (5x)^2$, and
$x = 8$.

27. To find two numbers such that their sum and product are given numbers.

Necessary condition. The square of half the sum must exceed the product by a square number. ἔστι δὲ τοῦτο πλασματικόν[1].

Given sum 20, given product 96.
$2x$ the difference of the required numbers.
Therefore the numbers are $10 + x$, $10 - x$.
Hence $100 - x^2 = 96$.
Therefore $x = 2$, and
the required numbers are **12, 8**.

28. To find two numbers such that their sum and the sum of their squares are given numbers.

Necessary condition. Double the sum of their squares must exceed the square of their sum by a square. ἔστι δὲ καὶ τοῦτο πλασματικόν[1].

[1] There has been controversy as to the meaning of this difficult phrase. Xylander, Bachet, Cossali, Schulz, Nesselmann, all discuss it. Xylander translated it by "effictum aliunde." Bachet of course rejects this, and, while leaving the word untranslated, maintains that it has an active rather than a passive signification; it is, he says, not something "made up" (effictum) but something "a quo aliud quippiam effingi et plasmari potest," "from which something else can be made up," and this he interprets as meaning that from the conditions to which the term is applied, combined with the solutions of the respective problems in which it occurs, the rules for solving mixed quadratics can be evolved. Of the two views I think Xylander's is nearer the mark. πλασματικὸν should apparently mean "of the nature of a πλάσμα," just as δραματικόν means something connected with or suitable for a drama; and πλάσμα means something

Given sum 20, given sum of squares 208.
Difference $2x$.
Therefore the numbers are $10+x$, $10-x$.
Thus $200+2x^2=208$, and $x=2$.
The required numbers are **12, 8.**

29. To find two numbers such that their sum and the difference of their squares are given numbers.

Given sum 20, given difference of squares 80.
Difference $2x$.
The numbers are therefore $10+x$, $10-x$.
Hence $(10+x)^2-(10-x)^2=80$,
or $40x=80$, and $x=2$.
The required numbers are **12, 8.**

30. To find two numbers such that their difference and product are given numbers.

Necessary condition. Four times the product together with the square of the difference must give a square. *ἔστι δὲ καὶ τοῦτο πλασματικόν.*

Given difference 4, given product 96.
$2x$ the sum of the required numbers.
Therefore the numbers are $x+2$, $x-2$; accordingly $x^2-4=96$, and $x=10$.
The required numbers are **12, 8.**

31. To find two numbers in a given ratio and such that the sum of their squares also has to their sum a given ratio.

Given ratios $3:1$ and $5:1$ respectively.
Lesser number x.
Therefore $10x^2=5.4x$, whence $x=2$, and
the numbers are **2, 6.**

32. To find two numbers in a given ratio and such that the sum of their squares also has to their difference a given ratio.

Given ratios $3:1$ and $10:1$.
Lesser number x, which is then found from the equation $10x^2=10.2x$.
Hence $x=2$, and
the numbers are **2, 6.**

"formed" or "moulded." Hence the expression would seem to mean "this is of the nature of a formula," with the implication that the formula is not difficult to make up or discover. Nesselmann, like Xylander, gives it much this meaning, translating it "das lässt sich aber bewerkstelligen." Tannery translates *πλασματικόν* by "formativum."