THE CATTLE-PROBLEM.

It is required to find the number of bulls and cows of each of four colours, or to find 8 unknown quantities. The first part of the problem connects the unknowns by seven simple equations; and the second part adds two more conditions to which the unknowns must be subject.

Let W, w be the numbers of white bulls and cows respectively,

X, x	„	„	black	„	„	„
Y, y	„	„	yellow	„	„	„
Z, z	„	„	dappled	„	„	„

First part.

(I)

$$W = (\tfrac{1}{2} + \tfrac{1}{3})\, X + Y \quad \ldots\ldots (\alpha),$$

$$X = (\tfrac{1}{4} + \tfrac{1}{5})\, Z + Y \quad \ldots\ldots (\beta),$$

$$Z = (\tfrac{1}{6} + \tfrac{1}{7})\, W + Y \quad \ldots\ldots (\gamma),$$

(II)

$$w = (\tfrac{1}{3} + \tfrac{1}{4})(X + x) \quad \ldots\ldots (\delta),$$

$$x = (\tfrac{1}{4} + \tfrac{1}{5})(Z + z) \quad \ldots\ldots (\epsilon),$$

$$z = (\tfrac{1}{5} + \tfrac{1}{6})(Y + y) \quad \ldots\ldots (\zeta),$$

$$y = (\tfrac{1}{6} + \tfrac{1}{7})(W + w) \quad \ldots\ldots (\eta).$$

Second part.

$$W + X = \text{a square} \quad \ldots\ldots (\theta),$$

$$Y + Z = \text{a triangular number} \quad \ldots\ldots (\iota).$$

[There is an ambiguity in the language which expresses the condition (θ). Literally the lines mean "When the white bulls joined in number with the black, they stood firm (ἔμπεδον) with depth and breadth of equal measurement (ἰσόμετροι εἰς βάθος εἰς εὖρός τε); and the plains of Thrinakia, far-stretching all ways, were filled with their multitude" (reading, with Krumbiegel, πλήθους instead of πλίνθου). Considering that, if the bulls were packed together so as to form a square *figure*, the number of them need not be a square *number*, since a bull is longer than it is broad, it is clear that one possible interpretation would be to take the 'square' to be a square *figure*, and to understand condition (θ) to be simply

$$W + X = \text{a rectangle (i.e. a product of two factors).}$$

The problem may therefore be stated in two forms:

(1) the simpler one in which, for the condition (θ), there is substituted the mere requirement that

$$W + X = \text{a product of two whole numbers;}$$

(2) the complete problem in which all the conditions have to be satisfied including the requirement (θ) that

$$W + X = \text{a square number.}$$

The simpler problem was solved by Jul. Fr. Wurm and may be called

Wurm's Problem.

The solution of this is given (together with a discussion of the complete problem) by Amthor in the *Zeitschrift für Math. u. Physik* (*Hist. litt. Abtheilung*), xxv. (1880), p. 156 sqq.

Multiply (α) by 336, (β) by 280, (γ) by 126, and add; thus

$$297\,W = 742\,Y, \text{ or } 3^3 . 11\,W = 2 . 7 . 53\,Y \quad\ldots\ldots\ldots(\alpha').$$

Then from (γ) and (β) we obtain

$$891Z = 1580Y, \text{ or } 3^4 . 11Z = 2^2 . 5 . 79Y \quad\ldots\ldots(\beta'),$$

and

$$99X = 178Y, \text{ or } 3^2 . 11X = 2 . 89Y \quad\ldots\ldots\ldots\ldots(\gamma').$$

Again, if we multiply (δ) by 4800, (ϵ) by 2800, (ζ) by 1260, (η) by 462, and add, we obtain

$$4657w = 2800X + 1260Z + 462Y + 143W;$$