Mathematics-Science 380 – History of Mathematics

TRENT UNIVERSITY, 2004–2005

Assignment #8 (Corrected)

Due 14 February, 2005.

As was noted in class, the poet and mathematician Omar al-Khayyami developed a geometric technique for finding the positive real roots of cubic and quartic equations. Recall that his method for solving cubics works as follows (in modern notation):

To solve the cubic equation

$$x^3 + ax^2 + b^2x + b^2c = 0,$$

intersect the hyperbola

$$y = \frac{bc}{x} + b$$

with the circle

$$\left(x + \frac{1}{2}(a+c)\right)^2 + y^2 = \frac{1}{4}(a-c)^2$$

and discard the point (-c, 0).

- 1. Verify that al-Khayyami's method for solving cubic equations actually does find positive real roots, if such exist. [10]
- 2. Find al-Khayyami's method for solving quartic equations or devise a geometrical method of your own for doing so, and verify that it works. [10]

A magic square is a square array of integers such that each column, each row, and each of the two diagonals add up to the same "magic" constant. For example, here is a 3×3 magic square:

The magic constant for this example is 15. Magic squares have been found in many cultures through the ages.

Bonus. Find as many occurrences of magic squares before 1500 A.D. as you can. [1]