Assignment #10
Due 14 March, 2005.

Goldbach's Conjecture is the assertion that every even number greater than or equal to four can be written as the sum of two (not necessarily different) prime numbers. For example, $4 = 2 + 2$, $6 = 3 + 3$, $8 = 5 + 3$, $10 = 7 + 3$ (or $10 = 5 + 5$), $12 = 7 + 5$, and so on. It's a conjecture rather than a theorem because no one has succeeded in proving it. (Or at least hasn't published!)

1. Give a brief sketch of Goldbach's life and career.   *[3]*

2. Write $65,794$ as a sum of two primes.   *[1]*

3. What results related to Goldbach's Conjecture have been proved? How close are they to the conjecture?   *[3]*

4. Suppose that we knew Goldbach's Conjecture could not be proved. Would that mean it wasn't true? Explain!   *[3]*

Bonus. Goldbach's Conjecture would have been readily comprehensible to classical Greek mathematicians. Did they make any conjectures about prime numbers that are still unsolved?   *[1]*

Lobachevsky alone has looked on Beauty bare.
She curves in here, she curves in here. She curves out there.
Her parallel clefts come together to tease
In un-callipygianous-wise;
With fewer than one hundred eighty degrees
Her glorious triangle lies.
Her double-trumpet symmetry Riemann did not court –
His tastes to simpler-curvedness, the buxom Teuton sort!
An ellipse is fine for as far as it goes,
But modesty, away!
If I'm going to see Beauty without her clothes
Give me hyperbolas any old day.

The world is curves, I've heard it said,
And straightway in it nothing lies.
This then my wish, before I'm dead:
To look through Lobachevsky's eyes.

This poem is by Roger Zelazny, from his novel *Doorways in the Sand*. It seems to be, at least in part, a parody or an homage — I can't quite decide! — of the following poem by Edna St. Vincent Millay.

Sonnet xlv

Euclid alone has looked on Beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminious air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light anatomized! Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.