

Mathematics 1121H – Calculus II

TRENT UNIVERSITY, Winter 2026

Final Examination

19:00-22:00 on Tuesday, 21 April, in the Gym.

Brought to you by Стефан Біланюк.

Instructions: Do parts **A**, **B**, and **C**, and, if you wish, part **D**. Show all your work and justify all your answers. *If in doubt about something, ask!*

Aids: One letter- or A4-size aid sheet with whatever you want on it, any calculator that can't communicate with other devices, one brain.

Part A. Do all three (3) of **1–3**. [60 = 3 × 20 each]

1. Evaluate any *four* (4) of the integrals **a–f**. [20 = 4 × 5 each]

$$\begin{array}{lll} \text{a. } \int \frac{1}{x^2 + 5x + 6} dx & \text{b. } \int_0^1 \left[\sum_{n=0}^{\infty} \frac{x^n}{n!} \right] dx & \text{c. } \int \frac{2 \sin(x) \cos(x)}{1 + \sin^4(x)} dx \\ \text{d. } \int_0^1 \frac{2x}{1 + x^4} dx & \text{e. } \int e^x \sin(x) dx & \text{f. } \int_{-\pi/8}^{\pi/8} \arctan(2x) dx \end{array}$$

2. Determine whether the series converges in any *four* (4) of **a–f**. [20 = 4 × 5 each]

$$\begin{array}{lll} \text{a. } \sum_{n=0}^{\infty} 2^n e^{-n} & \text{b. } \sum_{n=1}^{\infty} \frac{\ln(n)}{n^3} & \text{c. } \sum_{n=1}^{\infty} \frac{3^n + 1}{2^n - 1} \\ \text{d. } \sum_{n=0}^{\infty} \frac{(-1)^n n}{2n + 1} & \text{e. } \sum_{n=0}^{\infty} \frac{e^n}{n!} & \text{f. } \sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n + \sqrt{n} + 1} \end{array}$$

3. Do any *four* (4) of **a–g**. [20 = 4 × 5 each]

a. Find the arc-length of the curve $y = \frac{2}{3}x^{3/2}$, where $0 \leq x \leq 1$.

b. Does the series $\sum_{n=0}^{\infty} \frac{(-e)^n}{n!}$ converge absolutely, conditionally, or not at all?

c. Find the volume of the solid obtained by revolving the region between $y = 1 - x^2$ and $y = 0$, where $-1 \leq x \leq 1$, about the x -axis.

d. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{n^2 + 5n + 6}$.

e. Find the area of the finite region between $y = \cos(x)$ and $y = \sin(x)$, where $0 \leq x \leq 2\pi$.

f. What function of x has $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$ as its Taylor series at 0?

g. Find the Taylor series at 0 of $f(x) = \frac{d}{dx}(xe^x)$.

Parts **B–D** are on page 2.

Part B. Do any *two* (2) of **4–7**. [26 = 2 × 13 each]

4. **a.** Find the Taylor series at -1 of $g(x) = \sin(1 + x)$, [7]
b. and verify that it converges to $g(x)$ for all x . [6]
5. Consider the surface obtained by revolving the curve $y = \frac{x^2}{2}$, $0 \leq x \leq 2$, about the y -axis.
a. Sketch this surface. [3]
b. Find the area of this surface. [10]
6. Consider the power series $\sum_{n=0}^{\infty} (n+1)x^n = 1 + 2x + 3x^2 + 4x^3 + \dots$.
a. Determine the radius and interval of convergence of this power series. [8]
b. What function has this power series as its Taylor series? Why? [5]
7. Consider the region below $y = 2x$ and above $y = x^2$, for $0 \leq x \leq 2$.
a. Sketch the region and find its area. [3]
b. Sketch the solid obtained by revolving this region about the line $x = -1$ and find its volume. [10]

Part C. Do *one* (1) of **8** or **9**. [14]

8. Let $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ for $x > 0$. [This is the Gamma function.]
a. Show that $\Gamma(x+1) = x\Gamma(x)$ for all $x > 0$. [8]
b. Show that $\Gamma(1) = 1$. [3]
c. Explain why $\Gamma(n+1) = n!$ for all integers $n \geq 0$. [3]
9. Prove the p -Test: For $p \in \mathbb{R}$, $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$. [14]

[Total = 100]

Part D. Bonus problems! If you feel like it and have the time, do one or both of these.

- $\sqrt{100}$. A certain math student asserted that $1 + 2 + 4 + 8 + \dots = -1$. What exactly is the error they are likely to have made? [1]
- $\sqrt{121}$. Write a short poem touching on calculus or mathematics in general. [1]

ENJOY THE SUMMER!

P.S.: You may keep this question sheet. (It could be a souvenir, become a paper airplane, be used to help start a campfire, whatever! :-) The solutions to this exam will be posted to Blackboard and the course archive page in a day or three.