Mathematics 1121H – Calculus II
TRENT UNIVERSITY, Winter 2026

Assignment #11
Taylor Series
Due on Thursday, 2 April.[*]

1. Suppose a and r are real numbers. Find the Taylor series at 0 of $(a+x)^r$ and determine its radius of convergence. *[5]*

NOTE 1. You may find the following bit of notation handy. If r is a real number and $k \geq 1$ is an integer, then the *binomial of r and k* is

$$\binom{r}{k} = \frac{r(r-1)(r-2)\cdots(r-k+1)}{k!}\,.$$

Thus $\binom{r}{1} = r$, $\binom{r}{2} = \frac{r(r-1)}{2}$, $\binom{r}{3} = \frac{r(r-1)(r-2)}{6}$, and so on. Observe that $\binom{r}{k+1} = \frac{r-(k+1)+1}{k+1} \cdot \binom{r}{k} = \frac{r-k}{k+1} \cdot \binom{r}{k}$ for $k \geq 1$. To make various formulas work nicely, we let $\binom{r}{0} = 1$. Note that when r is a positive integer, this coincides with the usual definition of binomial coefficients.

NOTE 2. Since the Taylor series at 0 of $(a+x)^r$ turns out to be equal to the function inside the radius of convergence, this gives a result due to Isaac Newton (1642-1727) – yes, *that* Newton – that is nowadays called *Newton's Binomial Theorem*.

For question **2**, you may assume that $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, $\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$, and $\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, and that all three series converge for all real numbers x. Also, denote the square root of -1 by i, so $i^2 = -1$.

2. Prove *Euler's Formula*: $e^{ix} = \cos(x) + i\sin(x)$ for all real numbers x. *[3]*

NOTE. Plugging $x = \pi$ into Euler's Formula gives the equation $e^{i\pi} = -1$, which is also sometimes called Euler's Formula.

3. Use Euler's Formula to prove *de Moivre's Formula:* $(\cos(x) + i\sin(x))^n = \cos(nx) + i\sin(nx)$ for all real numbers x and integers n. *[2]*

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[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.