Assignment #8
Calculating π
Due just before midnight on Friday, 8 March.*

1. Verify that the series $\displaystyle\sum_{n=0}^{\infty} \frac{2}{(4n+1)(4n+3)}$ converges using one or more of the convergence tests given in class. *[2]*

2. Use SageMath to to find the sum of the series in **1**. *[1]*

3. What series involving powers of x has $\frac{1}{1+x^2}$ as its sum? For which values of x does this series converge? *[3]*

4. Since $\dfrac{d}{dx} \arctan(x) = \dfrac{1}{1+x^2}$ what series involving powers of x should be equal to $\arctan(x)$ when it converges? For which values of x does this series converge? *[3]*

Hint: This series converges for almost, but not quite, the same values of x that the series in **3** does.

5. Given that $\arctan(1) = \dfrac{\pi}{4}$, what is the connection betwen the series in **1** and **4**?

For One Who Loves An Engineer

Sing not to me of silicon chips
The chocolate kind are sweeter
But sweeter still would be your lips –
Put down that voltage meter!

A jug of wine (viscosity 3),
Loaf of bread (shear modulus 7)
But me you cannot quantify –
Oh, glory be to heaven!

Miriam Nadel

Mathematicians are like lovers. ... Grant a mathematician the least principle, and they will draw from it a consequence which you must also grant, and from this consequence another.

Bernard le Bovier Fontenelle