

Mathematics 1120H – Calculus II: Integrals and Series

TRENT UNIVERSITY, Winter 2024

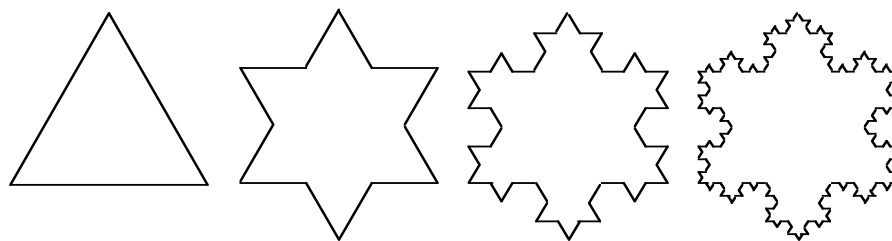
Assignment #7

A Snowflake Fence

Due\* just before midnight on Friday, 8 March.

A shape that looks something like a snowflake is defined by the following process:

Start with an equilateral triangle whose sides have length 1. If one modifies each of the line segments composing the triangle by cutting out the middle third of the segment, and then inserting an outward-pointing “tooth,” both of whose sides are as long as the removed third, one gets a six-pointed star. Suppose one repeats this process for each of the line segments making up the star, then to each of the line segments making up the resulting figure, and so on. The first few steps are illustrated below:



The snowflake shape is the limit of this process; *i.e.* what you have after infinitely many steps.

1. What is the length of the perimeter of the snowflake shape? [4]

*Hint:* How does the length of the perimeter change at each step of the process?

2. What is the area of the snowflake shape? [6]

*Hint:* What is the area of the original triangle? How much area is added at each step? Can you add up all these areas?

NOTE. You may find the following summation formula, already seen on Assignment #1, useful in doing 2. Recall that as long as  $|r| < 1$ :

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r}$$

The requirement that  $|r| < 1$  is necessary.

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\* You should submit your solutions via Blackboard’s Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.