Mathematics 1120H – Calculus II: Integrals and Series
Trent University, Winter 2024

Assignment #4
Area versus Area
Due just before midnight on Friday, 9 February.*

Consider the region below the curve $y = \dfrac{1}{x}$ and above the x-axis for $1 \leq x < \infty$, a piece of which you can colour in below.

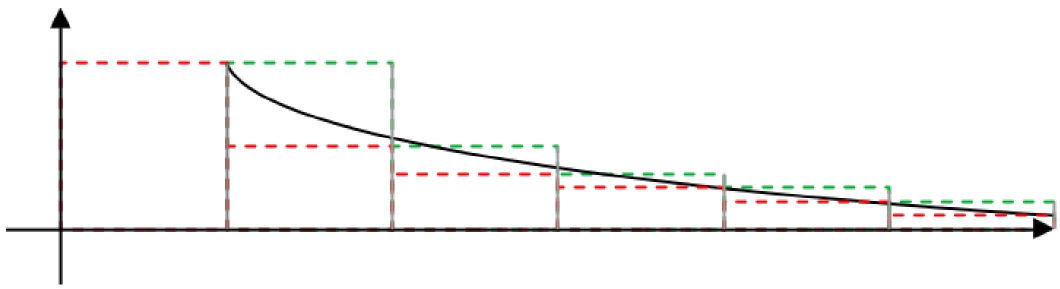
1. Compute each of the following as best you can using SageMath.

a. $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ *[0.5]* **b.** $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$ *[0.5]* **c.** $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ *[0.5]* **d.** $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2}$ *[0.5]*

Hint: It is possible that you might have done something similar to some of these previously. :-)

2. Explain why the sum $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$ is what it is because the integral $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ is what it is. *[2.5]*

Hint: A picture may be worth 10^3 words … Pay attention to the green (upper) dashed lines from 1 onward.



3. Explain why the sum $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2}$ has a finite value because the integral $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ has a finite value. *[2.5]*

Hint: Look at the picture above again. This time, pay attention to the red (lower, and between 0 and 1) dashed lines, especially from 1 onwards.

4. Explain why the limit $\displaystyle\lim_{k\to\infty}\left[\left(\sum_{n=1}^k \frac{1}{n}\right) - \ln(k+1)\right]$ exists and is between 0 and 1. *[2.5]*

Hint: Look at the picture yet again. This time, pay attention to the both the green (upper) and the red (lower) dashed lines from 1 onwards.

5. Use SageMath to (approximately) evaluate $\displaystyle\lim_{k\to\infty}\left[\left(\sum_{n=1}^k \frac{1}{n}\right) - \ln(k+1)\right]$ as best you can. *[0.5]*

Note: It is unknown whether the value of the limit in the last two questions is rational or irrational. If you can prove it one way or the other before the end of the term, your instructor will be *very* generous with your mark.

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.