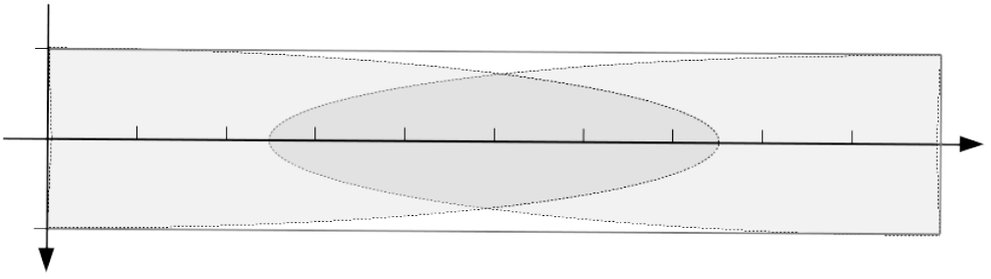
**Mathematics 1120H – Calculus II: Integrals and Series**
TRENT UNIVERSITY, Winter 2022

**Assignment #3**
**Definitely, let's integrate!**
*Due on Friday, 4 February. (May be submitted on paper or via Blackboard.*)*

Please show all your work. As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Consider the parabolas $y = a\left(1 - x^2\right)$ and $y = a\left(x^2 - 1\right) + 10$, where $a$ is a constant such that $0 \leq a \leq 10$, as in the picture below. (Rotate the page by a right angle counterclockwise to put the axes in the picture in a more familiar orientation. :-)



1. Find the area between the $x$-axis and the graph of $y = a\left(1 - x^2\right)$
   **a.** by hand, *[2]*
   **b.** and again by using `SageMath`. *[2]*

*Hint:* To do part **b**, try the `integral` operator. (See §1.12 in *Sage for Undergraduates*.) Of course, you can find harder ways to do it ... :-)

2. Find the value of the constant $a$ that would make the region which is both below the parabola $y = a\left(1 - x^2\right)$ and above the parabola $y = a\left(x^2 - 1\right) + 10$ have area equal to 5. *[6]*

*Hint:* This can be done by hand, though it will take a little while, but there may be some useful tools described in §1.8–1.9 of *Sage for Undergraduates*, at least when combined with the `integral` operator, if you would rather use `SageMath`.

NOTE: Any errors in this assignment are not intentional ...

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`