Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2020

Take-Home Final Examination
Released at noon on Tuesday, 14 April, 2020.
Due by noon on Friday, 17 April, 2020.

INSTRUCTIONS

- You may consult your notes, handouts, and textbook from this course and any other math courses you have taken or are taking now. You may also use a calculator. However, you may not consult any other source, or give or receive any other aid, except for asking the instructor to clarify instructions or questions.
- Please submit an electronic copy of your solutions, preferably as a single pdf (a scan of handwritten solutions should be fine), via the Assignment module on Blackboard. If that doesn't work, please email your solutions to the intructor.
- Do all three (3) of Parts **I** – **III**, and, if you wish, Part **IV** as well.

Part I. Do both of **1** and **2**. *[40 = 2×20 each]*

1. Compute the integrals in four (4) of **a** – **f**. *[20 = 4×5 each]*

a. $\displaystyle\int_0^{\pi/2} \cos(x)\sqrt{1+\sin^2(x)}\,dx$ **b.** $\displaystyle\int 2x^3e^{-x^2}\,dx$ **c.** $\displaystyle\int \frac{(x+1)^2}{x^2+1}\,dx$

d. $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^2(x)\cos^3(x)\,dx$ **e.** $\displaystyle\int_0^{\infty} xe^{-x}\,dx$ **f.** $\displaystyle\int e^x\cos(x)\,dx$

2. Determine whether the series converges or not in four (4) of **a** – **f**. *[20 = 4×5 each]*

a. $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{2^n n^n}$ **b.** $\displaystyle\sum_{n=0}^{\infty}3^n2^{-n}$ **c.** $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{e^n+n}$

d. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{\ln(n)}$ **e.** $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n\left[\ln(n)\right]^2}$ **f.** $\displaystyle\sum_{n=0}^{\infty}\frac{e^n}{e^{2n}+1}$

Part II. Do any two (2) of **3** – **5**. *[20 = 2×10 each]*

3. Find the volume of the solid obtained by revolving the region below $y = 4 - x^2$ and above $y = 0$, for $0 \le x \le 2$, about the y-axis. *[10]*

4. Find the centroid of the region outside the circle $x^2 + (y+4)^2 = 25$ and inside the circle $x^2 + y^2 = 9$. *[10]*

5. Find the area of the surface obtained by revolving the curve $y = 4 - x^2$, for $0 \le x \le 2$, about the y-axis. *[10]*

More exam on page 2!

Part III. Do any two (2) of **6** – **8**. *[20 = 2×10 each]*

6. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$. What function is it the Taylor series of? *[10]*

7. Suppose $p(x) = a_0 + a_1 x + a^2 x^2 + \cdots + a_k x^k$ is a polynomial of degree k. Find the Taylor series of $p(x)$, and find its radius and interval of convergence. *[10]*

8. Use the Taylor series of the three functions involved to show that $e^{ix} = \cos(x) + i\sin(x)$, where $i^2 = -1$, *i.e.* $i = \sqrt{-1}$. *[10]*

[Total = 80]

Part IV. Bonus! If you want to, do one or both of the following problems.

41. Write a poem touching on calculus or mathematics in general. *[1]*

42. Answer the riddle below, which supposedly gives the length of the Hellenistic mathematician Diophantus of Alexandria's life. *[1]*

126.—ΑΛΛΟ

Οὗτός τοι Διόφαντον ἔχει τάφος· ἆ μέγα θαῦμα·
καὶ τάφος ἐκ τέχνης μέτρα βίοιο λέγει.
ἕκτην κουρίζειν βιότου θεὸς ὤπασε μοίρην·
δωδεκάτην δ' ἐπιθείς, μῆλα πόρεν χνοάειν·
τῇ δ' ἄρ' ἐφ' ἑβδομάτῃ τὸ γαμήλιον ἥψατο φέγγος,
ἐκ δὲ γάμων πέμπτῳ παῖδ' ἐπένευσεν ἔτει.
αἰαῖ, τηλύγετον δειλὸν τέκος, ἥμισυ πατρὸς
†τοῦδε καὶ ἡ κρυερὸς μέτρον ἑλὼν βιότου.
πένθος δ' αὖ πισύρεσσι παρηγορέων ἐνιαυτοῖς
τῇδε πόσου σοφίῃ τέρμ' ἐπέρησε βίου.

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This tomb holds Diophantus. Ah, how great a marvel! the tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.

Metrodorus, Epigram 126, Greek Anthology

THANK YOU ALL FOR BEARING WITH THE COURSE
UNDER DIFFICULT CIRCUMSTANCES. IT HAS BEEN
BOTH A PLEASURE AND AN HONOUR TO TEACH YOU.
MAY YOU AND YOURS BE WELL AND SAFE, AND
MAY WE SEE EACH OTHER AGAIN IN BETTER TIMES.