Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Summer 2018

Assignment #5
Are you series?
Due on Wednesday, 25 July.

1. Suppose x is a variable and a_n for $n \geq 0$ are constants such that

$$\begin{aligned} \sum_{n=0}^{\infty} a_n x^n &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \\ &= \left(1 + x + x^2 + x^3 + \cdots\right)^2 = \left(\sum_{n=0}^{\infty} x^n\right)^2 . \end{aligned}$$

Find a formula for a_n in terms of n. *[4]*

HINT: Work out the first few a_ns by multiplying out $\left(1 + x + x^2 + x^3 + \cdots\right)^2$ and then collecting like terms, and look for a pattern.

2. It is a fact that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$, and the series in question converges for all $x \in \mathbb{R}$. There is another power series $\sum_{n=0}^{\infty} b_n x^n$ such that

$$\left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right)\left(\sum_{n=0}^{\infty} b_n x^n\right) = 1$$

for every value of the variable x. Find a formula for b_n in terms of n and determine for what values of x the series $\sum_{n=0}^{\infty} b_n x^n$ converges. *[6]*