Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

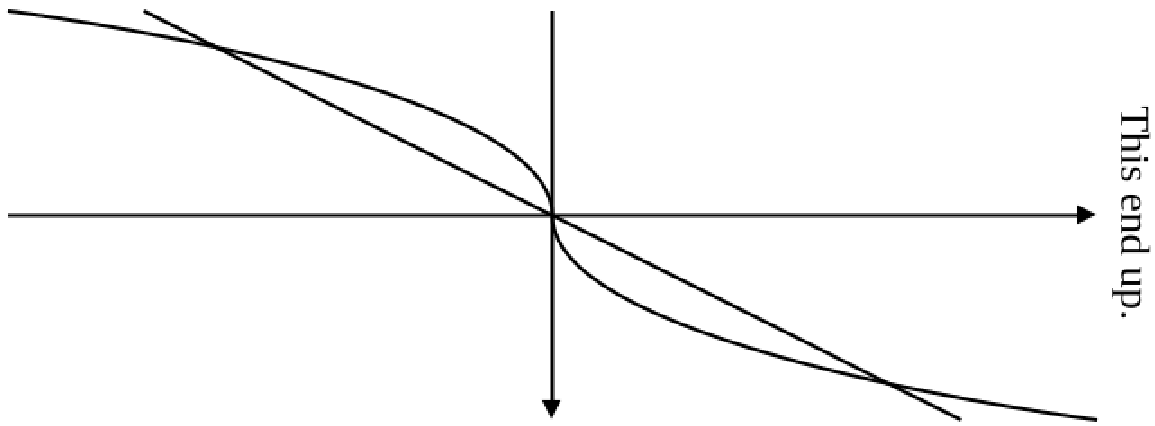
TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

Quiz #8

Due just before midnight on Friday, 17 July.

1. Consider the cubic $y = x^3$ and a line $y = mx$. If $m > 0$, then there is a finite region bounded above by the line and below by the cubic. Find the value of m that ensures that the area of this region is 4. *[5]*

SOLUTION. First, when do $y = x^3$ and $y = mx$ intersect? When $x^3 = y = mx$, *i.e.* when $x^3 - mx = 0$; that is, when $x\left(x^2 - m\right) = x\left(x - \sqrt{m}\right)\left(x + \sqrt{m}\right) = 0$, which happens when $x = 0$, $x = \sqrt{m}$ and when $x = -\sqrt{m}$. (Note that $\sqrt{m}$ makes sense when $m > 0$.) A quick look at a sketch of the setup ...



... is probably enough to tell us that the finite region where $y = mx$ is above $y = x^3$ starts when $x \geq 0$ and continues while $x \leq \sqrt{m}$.

Second, the area of this region is therefore

$$\int_0^{\sqrt{m}} (\text{upper} - \text{lower})\, dx = \int_0^{\sqrt{m}} \left(mx - x^3\right)\, dx = \left(\frac{mx^2}{2} - \frac{x^4}{4}\right)\Big|_0^{\sqrt{m}}$$
$$= \left(\frac{m\left(\sqrt{m}\right)^2}{2} - \frac{\left(\sqrt{m}\right)^4}{4}\right) - \left(\frac{m0^2}{2} - \frac{0^4}{4}\right)$$
$$= \frac{m^2}{2} - \frac{m^2}{4} = \frac{m^2}{4}.$$

Finally, we solve $\dfrac{m^2}{4} = 4$ for m:

$$\frac{m^2}{4} = 4 \implies m^2 = 4 \cdot 4 = 16 \implies m = \pm\sqrt{16} = \pm 4 \implies m = 4,$$

since we are given that $m > 0$. ■