Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

Solutions to Quiz #7

Due just before midnight on Wednesday, 15 July.

Compute both of the following integrals by hand, showing all the major steps.

1. $\displaystyle\int \tan(x)\sec(x)\,dx$ *[2]*

SOLUTION. *0. Substitute for* $\tan(x)$. If $u = \tan(x)$, then $du = \sec^2(x)\,dx$. Since we only have $\sec(x)$ in the integrand instead of $\sec^2(x)$, this is probably going nowhere … *Failure!* □

SOLUTION. *1. Substitute for* $\sec(x)$. If $w = \sec(x)$, then $dw = \sec(x)\tan(x)\,dx = \tan(x)\sec(x)\,dx$, so $\displaystyle\int \tan(x)\sec(x)\,dx = \int 1\,dw = w + C = \sec(x) + C$. *Success!* □

SOLUTION. *2. Solve the differential equation* $\dfrac{dy}{dx} = \tan(x)$. Do we know any functions that have derivative $\tan(x)\sec(x)$? Why yes: $y = \sec(x)$ does! Since addinga constant doesn't change the derivative, the general solution would be $y = \sec(x) + C$. ■

2. $\displaystyle\int_1^e x\,(\ln(x))^2\,dx$ *[3]*

SOLUTION. This is a job for integration by parts, with $u = (\ln(x))^2$ and $v' = x$, so $u' = 2\ln(x)\dfrac{1}{x}$ and $v = \dfrac{x^2}{2}$.

$$\begin{aligned}
\int_1^e x\,(\ln(x))^2\,dx &= \frac{x^2}{2}\,(\ln(x))^2\Big|_1^e - \int_1^e 2\ln(x)\frac{1}{x}\cdot\frac{x^2}{2}\,dx \\
&= \frac{e^2}{2}\,(\ln(e))^2 - \frac{1^2}{2}\,(\ln(1))^2 - \int_1^e x\ln(x)\,dx \\
&\qquad \text{Use parts again, with } s = \ln(x) \text{ and } t' = x \text{, so } s' = \frac{1}{x} \text{ and } t = \frac{x^2}{2}. \\
&= \frac{e^2}{2}1^2 - \frac{1}{2}0^2 - \left[\frac{x^2}{2}\ln(x)\Big|_1^e - \int_1^e \frac{1}{x}\cdot\frac{x^2}{2}\,dx\right] \\
&= \frac{e^2}{2} - 0 - \left(\frac{e^2}{2}\ln(e) - \frac{1^2}{2}\ln(1)\right) + \frac{1}{2}\int_1^e x\,dx \\
&= \frac{e^2}{2} - \left(\frac{e^2}{2}1 - \frac{1}{2}0\right) + \frac{1}{2}\cdot\frac{x^2}{2}\Big|_1^e \\
&= \frac{e^2}{2} - \frac{e^2}{2} + 0 + \frac{e^2}{4} - \frac{1^2}{4} = 0 + \frac{e^2}{4} - \frac{1}{4} = \frac{e^2-1}{4} \quad ■
\end{aligned}$$