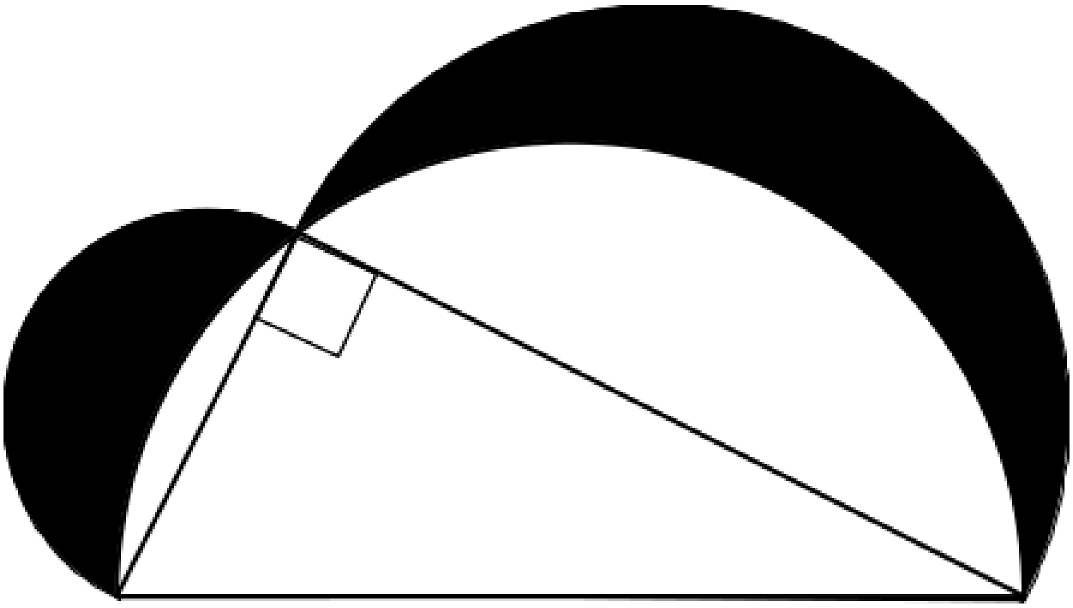
**Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals**

TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

**Solution to Quiz #5**

*Due just before midnight on Tuesday, 28 July. (The last day of classes.)*



A semicircle is placed on each of the sides of a right triangle with area 6, each having the given side of the triangle be the semicircle's base. The semicircles on the short sides are outside the triangle and the semicircle on the hypotenuse includes the triangle, as in the diagram above. (In fact, the vertex with the right angle must necessarily be on the latter semicircle.) Consider the region that is left over when the combined semicircles on the short sides have the parts that fall in the semicircle on the hypotenuse cut away; that is, the region that is shaded in the diagram.

**1.** What is the area of the shaded region? *[5]*

SOLUTION. The area of the shaded region in such a setup turns out to be equal to the area of the triangle, which is 6 in this case. (This result is due to Hippocrates of Chios (*c.* 470-420 A.D.)†.) Here's why:

Suppose the short sides of the triangle have lengths $a$ and $b$ and the hypotenuse has length $c$. The Pythagorean Theorem then tells us that $a^2+b^2=c^2$. Note that a semicircle with diameter $d$, and hence radius $\frac{d}{2}$, has area $\frac{1}{2}\pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{8}$, so it follows that the combined area of the two semicircles on the short sides of the triangle is equal to the area of the semicircle on the hypotenuse: $\dfrac{\pi a^2}{8}+\dfrac{\pi b^2}{8}=\dfrac{\pi}{8}\left(a^2+b^2\right)=\dfrac{\pi c^2}{8}$ It follows that:

$$\begin{aligned}\text{area of region} &= \text{area of triangle} + \text{combined area of semicircles on short sides}\\ &\qquad - \text{area of semicircle on hypotenuse}\\ &= \text{area of triangle} \quad \blacksquare\end{aligned}$$

† Not to be confused with his more famous contemporary, the physician Hippocrates of Kos (*c.* 460-370 A.D., after whom the Hippocratic Oath taken by medical doctors is named.