Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

Quiz #3

Due just before midnight on Friday, 26 June.

1. Work out the derivative of $\text{arcsec}(x)$, the inverse function of $\sec(x)$, showing all the major steps. *[2.5]*

SOLUTION. We will make use of the formula $\frac{d}{dx}f^{-1}(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$. Recall that $\frac{d}{dt}\sec(t) = \sec(t)\tan(t)$ and that $\tan^2(t) = \sec^2(t) - 1$, so $\tan(t) = \sqrt{\sec^2(t) - 1}$. Here we go:

$$\begin{aligned}\frac{d}{dx}\text{arcsec}(x) &= \frac{1}{\frac{d}{dt}sec(t)\Big|_{t=\text{arcsec}(x)}} = \frac{1}{\sec\left(\text{arcsec}(x)\right)\tan\left(\text{arcsec}(x)\right)} \\ &= \frac{1}{x\sqrt{\sec^2\left(\text{arcsec}(x)\right) - 1}} = \frac{1}{x\sqrt{x^2-1}} \quad \blacksquare\end{aligned}$$

2. Work out $\frac{dy}{dx}$ if $e^y = \frac{x+1}{x^3+1}$, showing all the major steps. *[2.5]*

HINT. There are several different ways to go about this one ...

SOLUTION 1. *Solve for y and proceed using the various differentiation rules.*

First, we solve for y: $y = \ln\left(e^y\right) = \ln\left(\frac{x+1}{x^3+1}\right)$.

Second, we differentiate away, using the Chain and Quotient Rules, among others:

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\ln\left(\frac{x+1}{x^3+1}\right) = \frac{1}{\frac{x+1}{x^3+1}} \cdot \frac{d}{dx}\left(\frac{x+1}{x^3+1}\right) \\ &= \frac{x^3+1}{x+1} \cdot \frac{\left[\frac{d}{dx}(x+1)\right]\left(x^3+1\right) - (x+1)\left[\frac{d}{dx}\left(x^3+1\right)\right]}{\left(x^3+1\right)^2} \\ &= \frac{1}{x+1} \cdot \frac{[1]\left(x^3+1\right) - (x+1)\left[3x^2\right]}{x^3+1} = \frac{x^3+1-3x^3-3x^2}{(x+1)\left(x^3+1\right)} \\ &= \frac{-2x^3-3x^2+1}{(x+1)\left(x^3+1\right)} = \frac{(x+1)\left(-2x^2+x+1\right)}{(x+1)\left(x^3+1\right)} = \frac{-2x^2+x+1}{x^3+1} \quad \square\end{aligned}$$

SOLUTION 2. *More use of logarithms.*

First, $y = \ln\left(e^y\right) = \ln\left(\frac{x+1}{x^3+1}\right) = \ln(x+1) - \ln\left(x^3+1\right)$.

Second, we differentiate:

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\ln(x+1) - \ln\left(x^3+1\right)\right) = \frac{1}{x+1} \cdot \frac{d}{dx}(x+1) - \frac{1}{x^3+1} \cdot \frac{d}{dx}\left(x^3+1\right) \\ &= \frac{1}{x+1} \cdot 1 - \frac{1}{x^3+1} \cdot 3x^2 = \frac{1}{x+1} - \frac{3x^2}{x^3+1} = \frac{x^2-x+1}{(x+1)\left(x^2-x+1\right)} - \frac{3x^2}{x^3+1} \\ &= \frac{x^2-x+1-3x^2}{x^3+1} = \frac{-2x^2+x+1}{x^3+1} \quad \square\end{aligned}$$

Solution 3. *Implicit differentiation.*

We differentiate both sides and see what happens.

$$\begin{aligned}
\frac{d}{dx}e^y = e^y\frac{dy}{dx} &= \left(\frac{x+1}{x^3+1}\right)\frac{dy}{dx} \quad \text{On the one hand} \ldots \\
&= \frac{d}{dx}\left(\frac{x+1}{x^3+1}\right) = \frac{d}{dx}\left(\frac{x+1}{x^3+1}\right) \\
&= \frac{\left[\frac{d}{dx}(x+1)\right]\left(x^3+1\right) - (x+1)\left[\frac{d}{dx}\left(x^3+1\right)\right]}{\left(x^3+1\right)^2} \\
&= \frac{[1]\left(x^3+1\right) - (x+1)\left[3x^2\right]}{\left(x^3+1\right)^2} = \frac{x^3+1-3x^3-3x^2}{\left(x^3+1\right)^2} \\
&= \frac{-2x^3-3x^2+1}{\left(x^3+1\right)^2} \qquad \ldots \text{ on the other.}
\end{aligned}$$

Thus $\left(\frac{x+1}{x^3+1}\right)\frac{dy}{dx} = \frac{-2x^3-3x^2+1}{\left(x^3+1\right)^2}$. It follows that

$$\begin{aligned}
\frac{dy}{dx} &= \frac{1}{\frac{x+1}{x^3+1}} \cdot \frac{-2x^3-3x^2+1}{\left(x^3+1\right)^2} = \frac{x^3+1}{x+1} \cdot \frac{-2x^3-3x^2+1}{\left(x^3+1\right)^2} \\
&= \frac{-2x^3-3x^2+1}{(x+1)\left(x^3+1\right)} = \frac{(x+1)\left(-2x^2+x+1\right)}{(x+1)\left(x^3+1\right)} = \frac{-2x^2+x+1}{x^3+1} \qquad \square
\end{aligned}$$

Note. One can obviously mix and match between the methods above. Note that the possibly hardest part is spotting that $x+1$ is a factor of $-2x^3-3x^2+1$ when simplifying the final answer.