Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2026 (S62)

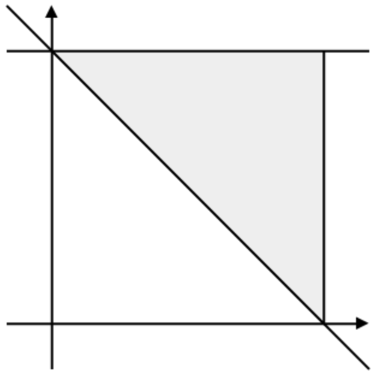
Quiz #10

*Due just before midnight on Friday, 24 July.**

Consider the region below the line $y = 3$ and above the line $y = 3 - x$, for $0 \leq x \leq 3$.

1. Sketch this region and find its area. *[1.5]*

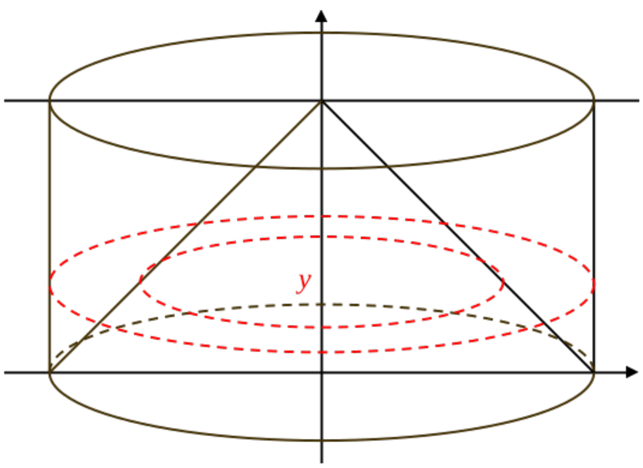
SOLUTION. Here's sketch of the region:



As the region is a triangle with base and height both equal to 3, it has area $\frac{bh}{2} = \frac{3 \cdot 3}{2} = \frac{9}{2} = 4.5$. ■

2. Sketch the solid obtained by revolving this region about the line $x = 0$ and find its volume. *[3.5]*

SOLUTION. *i. Disk/washer method.* Here's a sketch with a washer cross-section drawn in:



* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If that fails, please submit directly to the instructor on paper or via email to `sbilaniuk@trentu.ca` . You may work with others, use whatever tools you like, and look things up, so long as you write up your submission by yourself and give due credit to your collaborators and any tools or sources you actually used.

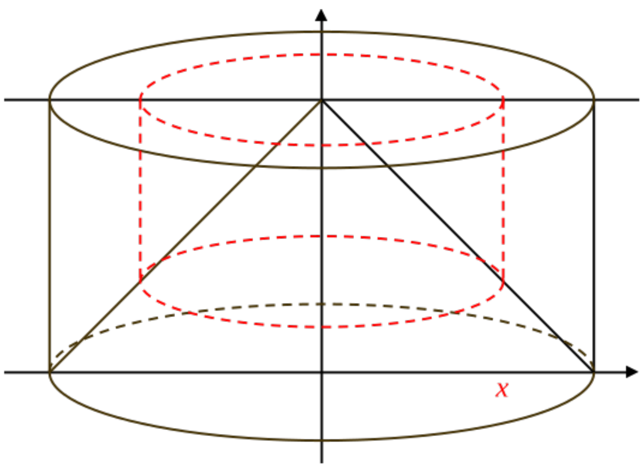
Since the given region was revolved about a vertical line, the washer cross-sections are parallel to the x-axis and perpendicular to the y-axis. The washer at y has outer radius $R = 3 - 0 = 3$ and inner radius $r = x - 0 = 3 - y$. (Note that $y = 3 - x \iff x = 3 - y$ and that $0 \le y \le 3$ over the region.) It follows that this cross-section has area

$$A(y) = \pi \left[R^2 - r^2\right] = \pi \left[3^2 - (3-y)^2\right] = \pi \left[9 - \left(9 - 6y + y^2\right)\right] = \pi \left[6y - y^2\right].$$

It follows, in turn, that the volume of the solid is

$$\begin{aligned} V &= \int_0^3 A(y)\,dy = \int_0^3 \pi \left[6y - y^2\right]\,dy = \pi \left[6 \cdot \frac{y^2}{2} - \frac{y^3}{3}\right]_0^3 = \pi \left[3y^2 - \frac{y^3}{3}\right]_0^3 \\ &= \pi \left[3 \cdot 3^2 - \frac{3^3}{3}\right] - \pi \left[3 \cdot 0^2 - \frac{0^3}{3}\right] = \pi\,[27 - 9] - \pi \cdot 0 = 18\pi. \quad \square \end{aligned}$$

ii. Cylindrical shell method. Here's a sketch with a cylindrical shell cross-section drawn in:



Since the given region was revolved about a vertical line, the cylindrical shells are parallel to the y-axis and perpendicular to the x-axis. The shell at x, where $0 \le x \le 3$, has height $h = 3 - (3 - x) = x$ and radius $r = x - 0 = x$, so this shell has area $A(x) = 2\pi r h = 2\pi x \cdot x = 2\pi x^2$. It follows, in turn, that the volume of the solid is

$$V = \int_0^3 A(x)\,dx = \int_0^3 2\pi x^2\,dx = 2\pi \frac{x^3}{3}\Big|_0^3 = 2\pi \frac{3^3}{3} - 2\pi \frac{0^3}{3} = 2\pi \cdot 9 - 2\pi \cdot 0 = 18\pi. \quad \blacksquare$$