Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

~~Mini-Assignment~~ Quiz #1

Due just before midnight on Friday, 19 June

1. Use the ε–δ definition of limits to verify that $\lim_{x \to -3} (2x+3) = -3$. *[2.5]*

NOTE. You may use the standard version of the definition or the alternate game form of the definition, as you prefer.

SOLUTION. We'll use the standard version of the ε–δ definition of limits. This means that to verify that $\lim_{x \to -3} (2x+3) = -3$, we need to check that for any $\varepsilon > 0$, we can find a $\delta > 0$, such that if $|x - (-3)| < \delta$, then $|(2x+3) - (-3)| < \varepsilon$.

Suppose we are given an $\varepsilon > 0$. As usual we try find a corresponding $\delta > 0$ by reverse-engineering:

$$\begin{aligned} |(2x+3) - (-3)| < \varepsilon &\iff |2x+3+3| < \varepsilon \\ &\iff |2x+6| < \varepsilon \\ &\iff |2(x+3)| < \varepsilon \\ &\iff |x+3| < \frac{\varepsilon}{2} \\ &\iff |x-(-3)| < \frac{\varepsilon}{2} \end{aligned}$$

It follows that $\delta = \frac{\varepsilon}{2}$ will serve. For if $|x - (-3)| < \delta = \frac{\varepsilon}{2}$, we can follow the steps above in reverse – since each step is fully reversible – to obtain $|(2x+3) - (-3)| < \varepsilon$, as required.

Thus $\lim_{x \to -3} (2x+3) = -3$ by the standard version of the ε–δ definition of limits. ■

2. Work out $\lim_{t \to 1} \frac{t-1}{t^4-1}$ using the practical rules for computing limits, as codified in the Theorems of Section 2.3. *[2.5]*

Hint: Algebra!

SOLUTION. We'll use "difference of squares" twice, and a bit of cancellation.

$$\begin{aligned} \lim_{t \to 1} \frac{t-1}{t^4-1} &= \lim_{t \to 1} \frac{t-1}{\left(t^2\right)^2 - 1} = \lim_{t \to 1} \frac{t-1}{\left(t^2-1\right)\left(t^2+1\right)} = \lim_{t \to 1} \frac{t-1}{(t-1)(t+1)\left(t^2+1\right)} \\ &= \lim_{t \to 1} \frac{1}{(t+1)\left(t^2+1\right)} = \frac{1}{(1+1)\left(1^2+1\right)} = \frac{1}{2 \cdot 2} = \frac{1}{4} \quad ■ \end{aligned}$$

NOTE. We can't just try to evaluate the original function at $t = 1$: $1^4 - 1 = 1 - 1 = 0$, so we'd be dividing by 0, which is a no-no ...