Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
Trent University, Summer 2026 (S62)
Final Examination

09:00-12:00 on Thursday, 30 July, in ENW 117. *Brought to you by* Стефан Біланюк.

Instructions: Do parts **A** and **B**, and, if you wish, part **C**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part A. Do all four (4) of **1**–**4**.

1. Compute $\frac{dy}{dx}$ as best you can in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = e^{x^2+x}$ **b.** $x^3 - y^3 = 0$ **c.** $y = \int_1^{\sqrt{x}} t^2\,dt$

d. $y = \frac{1}{x^2+1}$ **e.** $y = e^x \sin(x)$ **f.** $y = \sec^2(x)$

2. Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\int x^2 \ln(x)\,dx$ **b.** $\int_0^{\pi/2} \sin(2x)\,dx$ **c.** $\int_e^{e^e} \frac{1}{x\ln(x)}\,dx$

d. $\int \frac{1}{(3x+2)^2}\,dy$ **e.** $\int x\cos(x)\,dx$ **f.** $\int_0^1 6x^2 e^{x^3}\,dx$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Find the equation of the tangent line to $y = \cos(x)$ at $x = \frac{\pi}{2}$.

b. Compute $\lim_{x\to\infty} \frac{e^x}{x^2}$.

c. Use the limit definition of the derivative to verify that $\frac{d}{dx}x^2 = 2x$ for all x.

d. Find the absolute maximum value of $f(x) = xe^{-x}$, if it has one.

e. Use the ε–δ definition of limits to verify that $\lim_{x\to 3}(2x-4) = 2$.

f. Sketch the region between $y = x^2$ and $y = 2x$ for $0 \le x \le 2$ and find its area.

4. Find the domain and all intercepts, vertical and horizontal asymptotes, intervals of increase and decrease and maximum and minimum points, and intervals of concavity and inflection points of $f(x) = e^{-x^2}$, and sketch its graph. *[14]*

Part B. Do any *two* (2) of **5–7**. *[28 = 2 × 13 each]*

5. A spherical balloon is being inflated at a rate of 1 m^3/s. How is its surface area changing at the instant that its volume is 36 m^3? *[13]*

[Recall that a sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.]

6. Consider the region below $y = x^2$ and above $y = 0$, where $0 \leq x \leq 9$.

a. Sketch the region and find its area. *[4]*

b. Sketch the solid obtained by revolving the region about the y-axis and find its volume. *[9]*

7. What is the maximum area of a triangle whose vertices are the points $(0, 0)$, $(x, 0)$, and $\left(x, e^{-x}\right)$ for some $x \geq 0$?

[Total = 100]

Part C. Bonus problems! If you feel like it and have the time, do one or both of these.

△. Suppose a number of circles are drawn on a piece of paper, dividing it up into regions whose borders are made up of circular arcs. Show that one can always colour these regions using only black and white so that no two regions that have a border arc in common have the same colour. *[1]*

□. Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

I HOPE THAT YOU ENJOYED THE COURSE.
ENJOY THE REST OF THE SUMMER!