MATH1110H-S62-A6-Solutions-SageMath

July 29, 2026

[1]:
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# MATH 1110H,  Summer 2026 (S62)
# Solutions to Assignment #6
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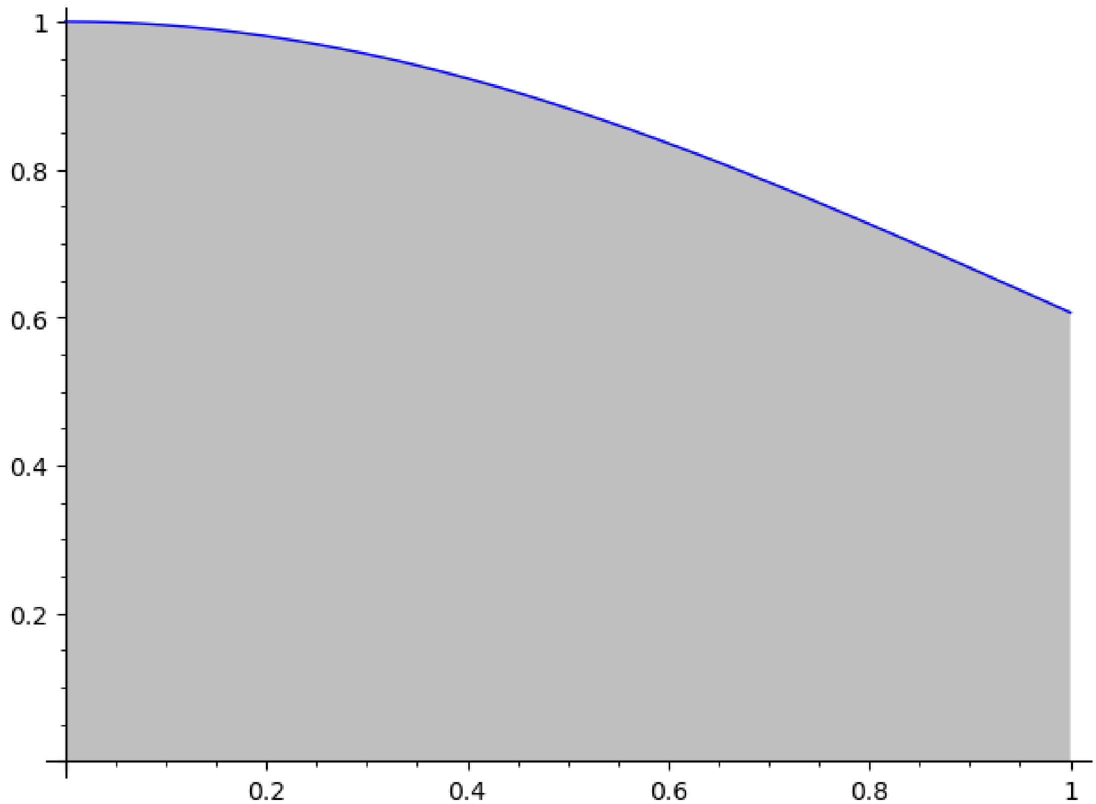
[2]:
```
# 1
# Here is a plot of the region below y = e^(-x^2/2) and above y=0...
plot( e^(-x^2/2), 0, 1, fill=True )
```

[2]:



[3]:
```
# 1 continued
# ... and here is its area.
integral( e^(-x^2/2), x, 0, 1 )
```

[3]: 1/2*sqrt(2)*sqrt(pi)*erf(1/2*sqrt(2))

[4]:
```
# 1 continued some more
# The above isn't very illuminating,  so we convert it to a decimal.
N(1/2*sqrt(2)*sqrt(pi)*erf(1/2*sqrt(2)))
```

[4]: 0.855624391892149

[5]:
```
# 2
# See the appended document for a sketch of the solid obtained by
# revolving the region about the y-axis.  If we set up the volume
# integral for this solid using the method of cylindrical shells,
# the height of the shell at x is h = y-0 = e^(-x^2/2) and its
# radius is r = x-0 = x,  giving us the following integral:
integral( 2*pi*x*e^(-x^2/2), x, 0, 1 )
# This is the one integral in this assignment that you should be
# able to do by hand without struggling...
```

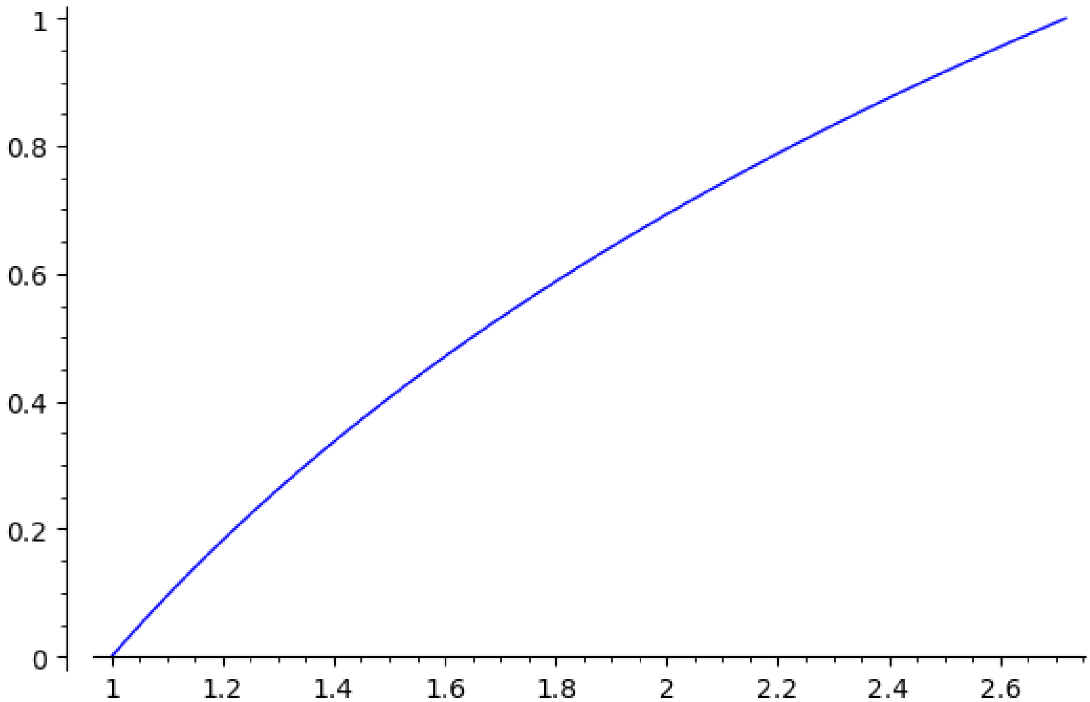
[5]: -2*pi*(e^(-1/2) - 1)

[6]:
```
# 2 continued
# For those who like their decimal approximations:
N(-2*pi*(e^(-1/2) - 1))
```

[6]: 2.47224077771923

[7]:
```
# 3
# Here is a plot of the curve y = ln(x),  1<= x <= e.
plot( log(x), 1, e )
```

[7]:



[8]:
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# 3 continued
show(diff( log(x), x ))
# Plugging dy/dx into the arc-length integral formula gives us:
integral( sqrt(1+(1/x)^2), x, 1, e )
```

1/x

[8]: -sqrt(2) + sqrt(e^2 + 1) + 1/2*log(sqrt(2) + 1) - 1/2*log(sqrt(2) - 1) - 1/2*log(sqrt(e^2 + 1) + 1) + 1/2*log(sqrt(e^2 + 1) - 1)

[9]:
```
# 3 continued
# This also isn't very illuminating,  so we convert it to a decimal.
N(-sqrt(2) + sqrt(e^2 + 1) + 1/2*log(sqrt(2) + 1) - 1/2*log(sqrt(2) - 1) - 1/
↪2*log(sqrt(e^2 + 1) + 1) + 1/2*log(sqrt(e^2 + 1) - 1))
```

[9]: 2.00349711162735

[10]:
```
# 4
# See the appended document for a sketch of the surface obtained by
# revolving the curve y = ln(x),  1<= x <= e,  about the y-axis.
# The point on the curve at x travels through a circle with radius
# r = x-0 = x.  Plugging this and dy/dx = 1/x into the surface area
# integral formula gives us:
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```
integral( 2*pi*x*sqrt(1+(1/x)^2), x, 1, e )
```

[10]:
```
1/2*pi*(2*sqrt(e^2 + 1)*e - 2*sqrt(2) + log(sqrt(e^2 + 1)*e^(-1) + 1) -
log(sqrt(e^2 + 1)*e^(-1) - 1) - log(sqrt(2) + 1) + log(sqrt(2) - 1))
```

[11]:
```
# 4 continued
# This too is not very illuminating,  so we convert it to a decimal.
N(1/2*pi*(2*sqrt(e^2 + 1)*e - 2*sqrt(2) + log(sqrt(e^2 + 1)*e^(-1) + 1) -␣
↪log(sqrt(e^2 + 1)*e^(-1) - 1) - log(sqrt(2) + 1) + log(sqrt(2) - 1)))
```

[11]:
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22.9430223411722
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[ ]:
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Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

Assignment #6

Due on Friday, 31 July.
Please note that this is the day after the exam,
so **late submissions will not be accepted**.

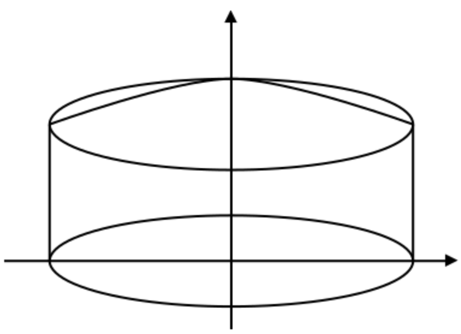
Please at least skim through Sections 9.3, 9.9, and 9.10 of our textbook, or else review the lectures on computing volumes of solids of revolution, arc-lengths, and areas of surfaces of revolution. Also, please take a look at Section 1.12 of Gregory Bard's *Sage for Undergraduates*.

Consider the region below $y = e^{-x^2/2}$ and above $y = 0$, for $0 \leq x \leq 1$.

1. Sketch or plot this region. What is the area of this region? *[2]*

2. Sketch the solid obtained by revolving this region about the y-axis. What is the volume of this solid? *[3]*

SOLUTION. Here is the desired sketch:



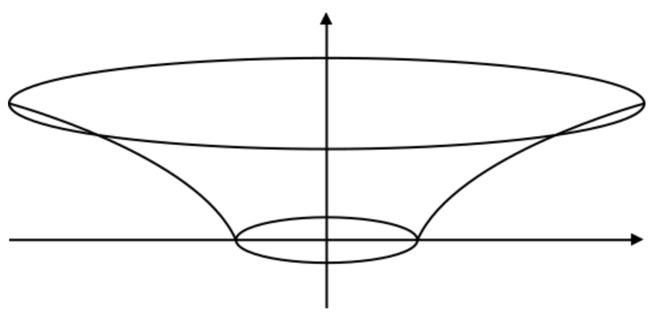
□

Consider the curve $y = \ln(x)$, for $1 \leq x \leq e$.

3. Sketch or plot this curve. What is the arc-length of this curve? *[2]*

4. Sketch the surface obtained by revolving this curve about the y-axis. What is the area of this surface? *[3]*

SOLUTION. Here is the desired sketch:



□

NOTE. Out of the four integrals you are asked to compute above, one is doable by hand with the integral techniques we know so far ...