Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2026 (S62) [reading course]

Assignment #4
Due on Friday, 17 July.

Please take a look at Sections 1.11 and 4.22 of Gregory Bard's *Sage for Undergraduates* before tackling this assignment.

1. Consider the differential equation $x\dfrac{dy}{dx} - y = 0$.

a. Use SageMath to find a general solution to this differential equation. *[1]*

b. Use SageMath to find a solution to this differential equation satisfying the initial condition $y = 1$ when $x = 1$. How many such solutions are there? *[1.5]*

c. Verify – by hand! – that any solutions you obtained in part **b** satisfy the given differential equation with the given initial conditions. *[1]*

SOLUTIONS. **a.** See the appended SageMath file. Note that the general solution is $y = cx$, where c could be any constant. $\square$

b. See the appended SageMath file. Note that there is only one solution to the given differential equation such that $y = 1$ when $x = 1$, namely $y = x$. $\square$

c. If $y = x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}x = 1$, so $x\dfrac{dy}{dx} - y = x \cdot 1 - x = 0$, satisfying the given differential equation. Also, when $x = 1$, $y = x = 1$, so the given initial condition is also satisfied. $\blacksquare$

2. Consider the differential equation $y\dfrac{dy}{dx} - x = 0$.

a. Use SageMath to find a general solution to this differential equation. *[1]*

b. Use SageMath to find a solution to this differential equation satisfying the initial condition $y = 0$ when $x = 0$. How many such solutions are there? *[1.5]*

c. Verify – by hand! – that any solutions you obtained in part **b** satisfy the given differential equation with the given initial conditions. *[1]*

SOLUTIONS. **a.** See the appended SageMath file. Note that the general solutions are $y = -\sqrt{x^2 + 2c}$ and $y = +\sqrt{x^2 + 2c}$, where c could be any constant. $\square$

b. See the appended SageMath file. Note that there are two solutions to the given differential equation such that $y = 0$ when $x = 0$, namely $y = -x$ and $y = +x$. $\square$

c. If $y = -x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(-x) = -1$, so $y\dfrac{dy}{dx} - x = -x \cdot (-1) - x = x - x = 0$, satisfying the given differential equation. Also, when $x = 0$, $y = -x = -0 = 0$, so the given initial condition is also satisfied.

If $y = x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}x = 1$, so $y\dfrac{dy}{dx} - x = x \cdot 1 - x = 0$, satisfying the given differential equation. Also, when $x = 0$, $y = x = 0$, so the given initial condition is also satisfied. $\blacksquare$

3. Find all the general solutions to the differential equation

$$xy\left(\frac{dy}{dx}\right)^2 - (x^2+y^2)\frac{dy}{dx} + xy = 0.$$

Explain why the solutions you found work. *[3]*

Hint: A little algebra can make this much easier.

Solution. See the appended SageMath file. Note that the general solutions to this differential equations are the general solutions to the differential equations in questions **1** and **2**. This because the given differential equation is the product of the differential equations in **1** and **2**,

$$xy\left(\frac{dy}{dx}\right)^2 - (x^2+y^2)\frac{dy}{dx} + xy = \left(x\frac{dy}{dx} - y\right)\left(y\frac{dy}{dx} - x\right) = 0 \cdot 0 = 0,$$

and the fact that a product is 0 if and only if at least one of the components of the product is 0. ∎

MATH1110H-S62-A4-Solutions-Sage

July 12, 2026

[1]:
```
# MATH 1110H,  Summer 2026 (S62)
# Solutions to the SageMath parts of Assignment #4
#
y = function('y')(x)    # We declare y to be a function so that we can
                        # safely differentiate it.
```

[2]:
```
# 1a
desolve( x*diff(y,x) - y == 0, y )
```

[2]: _C*x

[3]:
```
# 1b
desolve( x*diff(y,x) - y == 0, y, ics=[1,1] )
```

[3]: x

[4]:
```
# 2a
desolve( y*diff(y,x) - x == 0, y )
```

[4]: 1/2*y(x)^2 == 1/2*x^2 + _C

[5]:
```
# 2a continued
var('c')
solve( 1/2*y(x)^2 == 1/2*x^2 + c, y )
```

[5]: [y(x) == -sqrt(x^2 + 2*c), y(x) == sqrt(x^2 + 2*c)]

[6]:
```
# 2b
desolve( y*diff(y,x) - x == 0, y, ics=[0,0] )
```

[6]: 1/2*y(x)^2 == 1/2*x^2

[7]:
```
# 2b continued
solve( 1/2*y(x)^2 == 1/2*x^2, y )
```

[7]: [y(x) == -x, y(x) == x]

[8]:
```
# 3
desolve( x*y*(diff(y,x))^2 - (x^2 + y^2)*diff(y,x) + x*y == 0, y,␣
↪contrib_ode=True )
# Without the contrib_ode=True option you get an error message suggesting␣
↪setting
# the contrib_ode option to True.
```

[8]:
```
[y(x) == _C*x, 1/2*y(x)^2 == 1/2*x^2 + _C]
```

[9]:
```
# Notice that these are the solutions obtained in 1a and 2a,  respectively.
# This happens because the given differential equation is the product of the
# equations in 1a and 2a,  and because the only way for a product to be 0 is
# for at least one part of the product to be 0.
```