Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Trent University, Summer 2026 (S62) [reading course]

Solutions to Assignment #3

Due on Friday, 10 July.

Please read Section 1.8.1 of *Sage for Undergraduates*, which gives the basics of the `solve` command.

Recall that the hyperbolic sine function is $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$. We will call its inverse function $\operatorname{arcsinh}(x)$ and work out what it is in this assignment.

1. Plot $\sinh(x)$ for $-3 \leq x \leq 3$. On the basis of this graph, what should the domain of $\operatorname{arcsinh}(x)$ be? *[1.5]*

Solution. See the SageMath notebook appended below for the solution. $\square$

2. Use SageMath to work out $\operatorname{arcsinh}(x)$. *[4]*

Hint. Solve the equation $x = \dfrac{e^y - e^{-y}}{2}$ for y.

Solution. Note that $x = \sinh(y) = \dfrac{e^y - e^{-y}}{2}$ if and only if $y = \operatorname{arcsinh}(x)$. See the SageMath notebook appended below for the solution. $\square$

3. Work out $\operatorname{arcsinh}(x)$ by hand, showing all the principal steps. *[4]*

Solution. We follow the hint for question **2** again, except we proceed to solve $x = \sinh(y) = \dfrac{e^y - e^{-y}}{2}$ for y by hand, with a starring role for the quadratic formula along the way.

$$\begin{aligned}
x = \frac{e^y - e^{-y}}{2} &\iff 2x = e^y - \frac{1}{e^y} \iff 2xe^y = \left(e^y\right)^2 - 1 \iff \left(e^y\right)^2 - 2xe^y - 1 = 0 \\
&\iff e^y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} \\
&\iff e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = \frac{2x \pm 2\sqrt{x^2 + 1}}{2} = x \pm \sqrt{x^2 + 1} \\
&\iff y = \ln\left(e^y\right) = \ln\left(x \pm \sqrt{x^2 + 1}\right)
\end{aligned}$$

That is, there are two possibilities for $\operatorname{arcsinh}(x)$: $\operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right)$ and $\operatorname{arcsinh}(x) = \ln\left(x - \sqrt{x^2+1}\right)$. Note that these are the same as the possibilities given by SageMath in answering question **2**. Since $\ln(t)$ is undefined for $t < 0$ and $x - \sqrt{x^2+1} < 0$ for all x, the latter possibility for $\operatorname{arcsinh}(x)$ simply doesn't work. Thus $\operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right)$. $\square$

4. What is the domain of $\operatorname{arcsinh}(x)$? *[0.5]*

Solution. Since $|x| = \sqrt{x^2} < \sqrt{x^2+1}$ for all x, it follows that $x + \sqrt{x^2+1} > 0$ for all x. Since $\ln(t)$ is defined for all $t > 0$, it follows that $\operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right)$ is defined for all x, *i.e.* the domain of $\operatorname{arcsinh}(x)$ is all of $\mathbb{R} = (-\infty, \infty)$. $\blacksquare$

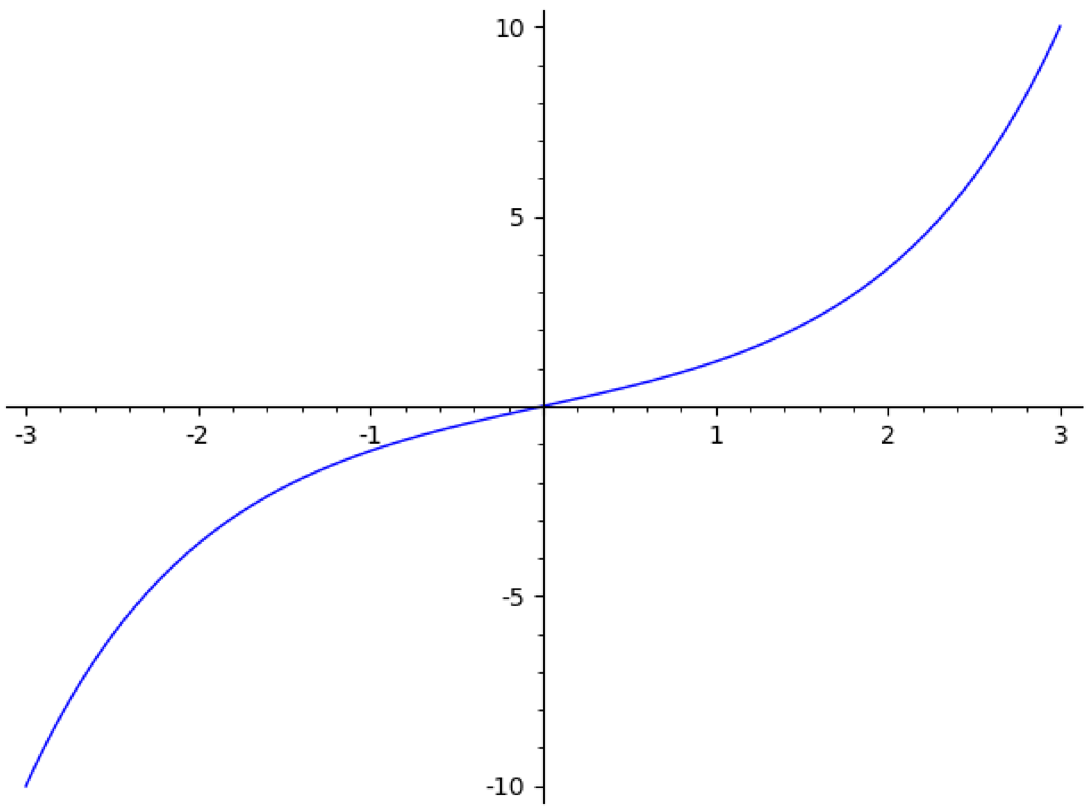
MATH1110H-S62-A3-Solutions-Sage

July 12, 2026

[1]:
```
# MATH 1110H Summer 2026 (S62)
# Solutions to the SageMath parts of Assignment #3
#
var('y')                    # We'll need it in Question 2.
s = function('s')(x)        # This is what we'll call sinh...
s(x) = (e^x - e^(-x))/2     # ... and this is its definition.
```

[2]:
```
# Question 1
plot( s(x), -3, 3 )
```

[2]:



[3]:
```
# Since every horizontal line intersects the graph exactly once,  the
# domain of the inverse function is all real numbers,  i.e. (-oo,oo).
```

[4]:
```
# Question 2
solve( x == s(y), y )  # So y will be the inverse of sinh.
```

[4]:
```
[y == log(x - sqrt(x^2 + 1)), y == log(x + sqrt(x^2 + 1))]
```

[ ]:
```

```