Solutions to Quiz #9 – How many parts make the whole?
Due on Tuesday, 22 July.

1. Compute $\int x^2 \cos(x)\, dx$ by hand, showing all the major steps. *[2]*

SOLUTION. We will use integration by parts twice. The first time, the parts will be $u = x^2$ and $v' = \cos(x)$, so $u' = 2x$ and $v = \sin(x)$.

$$\int x^2 \cos(x)\, dx = x^2 \sin(x) - \int 2x \sin(x)\, dx \quad \begin{array}{l} \text{Now we use the parts} \\ s = 2x \text{ and } t' = \sin(x), \text{ so} \\ s' = 2 \text{ and } t = -\cos(x). \end{array}$$

$$= x^2 \sin(x) - \left[2x\left(-\cos(x)\right) - \int 2\left(-\cos(x)\right)\, dx \right]$$

$$= x^2 \sin(x) + 2x \cos(x) - 2 \int \cos(x)\, dx$$

$$= x^2 \sin(x) + 2x \cos(x) - 2\sin(x) + C \quad \blacksquare$$

2. Compute $\int_1^e \left(\ln(x)\right)^2 dx$ by hand, showing all the major steps. *[3]*

SOLUTION. We will use integration by parts twice again. The first time, the parts will be $u = \left(\ln(x)\right)^2$ and $v' = 1$, so $u' = 2\ln(x) \cdot \dfrac{1}{x} = \dfrac{2\ln(x)}{x}$ and $v = x$.

$$\int_1^e \left(\ln(x)\right)^2 dx = x\left(\ln(x)\right)^2 \Big|_1^e - \int_1^e \frac{2\ln(x)}{x} \cdot x\, dx = x\left(\ln(x)\right)^2 \Big|_1^e - \int_1^e 2\ln(x)\, dx$$

Now we use the parts $s = 2\ln(x)$ and $t' = 1$, so $s' = \dfrac{2}{x}$ and $t = x$.

$$= \left[e\left(\ln(e)\right)^2 - 1\left(\ln(1)\right)^2 \right] - \left[2x\ln(x)|_1^e - \int_1^e \frac{2}{x} \cdot x\, dx \right]$$

$$= \left[e \cdot 1^2 - 1 \cdot 0^2 \right] - \left[\left(2e\ln(e) - 2 \cdot 1 \cdot \ln(1)\right) - \int_1^e 2\, dx \right]$$

$$= e - 0 - \left[\left(2e \cdot 1 - 2 \cdot 0\right) - 2x|_1^e \right] = e - \left[\left(2e - 0\right) - \left(2e - 2 \cdot 1\right) \right]$$

$$= e - \left[2e - 2e + 2 \right] = e - 2 \quad \blacksquare$$