

Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2025 (S62)

Solutions to Quiz #9 – How many parts make the whole?

Due on Tuesday, 22 July.

1. Compute $\int x^2 \cos(x) dx$ by hand, showing all the major steps. [2]

SOLUTION. We will use integration by parts twice. The first time, the parts will be $u = x^2$ and $v' = \cos(x)$, so $u' = 2x$ and $v = \sin(x)$.

$$\begin{aligned} \int x^2 \cos(x) dx &= x^2 \sin(x) - \int 2x \sin(x) dx && \begin{array}{l} \text{Now we use the parts} \\ s = 2x \text{ and } t' = \sin(x), \text{ so} \\ s' = 2 \text{ and } t = -\cos(x). \end{array} \\ &= x^2 \sin(x) - \left[2x(-\cos(x)) - \int 2(-\cos(x)) dx \right] \\ &= x^2 \sin(x) + 2x \cos(x) - 2 \int \cos(x) dx \\ &= x^2 \sin(x) + 2x \cos(x) - 2 \sin(x) + C \quad \blacksquare \end{aligned}$$

2. Compute $\int_1^e (\ln(x))^2 dx$ by hand, showing all the major steps. [3]

SOLUTION. We will use integration by parts twice again. The first time, the parts will be $u = (\ln(x))^2$ and $v' = 1$, so $u' = 2\ln(x) \cdot \frac{1}{x} = \frac{2\ln(x)}{x}$ and $v = x$.

$$\begin{aligned} \int_1^e (\ln(x))^2 dx &= x (\ln(x))^2 \Big|_1^e - \int_1^e \frac{2\ln(x)}{x} \cdot x dx = x (\ln(x))^2 \Big|_1^e - \int_1^e 2\ln(x) dx \\ &\quad \text{Now we use the parts } s = 2\ln(x) \text{ and } t' = 1, \text{ so } s' = \frac{2}{x} \text{ and } t = x. \\ &= \left[e (\ln(e))^2 - 1 (\ln(1))^2 \right] - \left[2x \ln(x) \Big|_1^e - \int_1^e \frac{2}{x} \cdot x dx \right] \\ &= \left[e \cdot 1^2 - 1 \cdot 0^2 \right] - \left[(2e \ln(e) - 2 \cdot 1 \cdot \ln(1)) - \int_1^e 2 dx \right] \\ &= e - 0 - [(2e \cdot 1 - 2 \cdot 0) - 2x \Big|_1^e] = e - [(2e - 0) - (2e - 2 \cdot 1)] \\ &= e - [2e - 2e + 2] = e - 2 \quad \blacksquare \end{aligned}$$