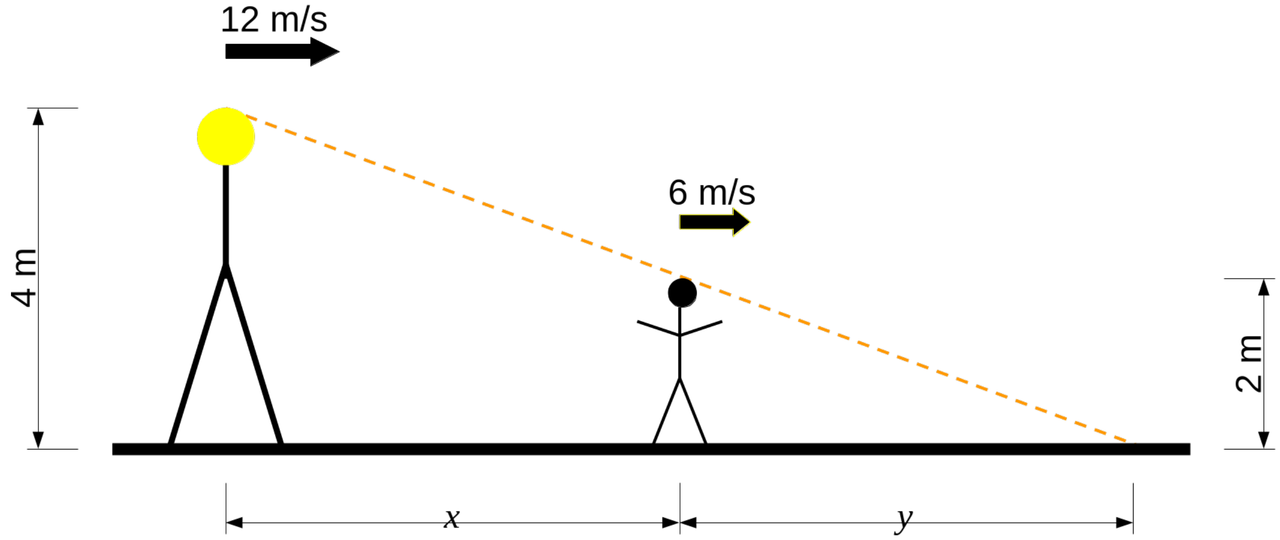
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2025 (S62)

Solutions to Quiz #7 – Related Rates Redux
Due on Tuesday, 15 July.

On the Island of Light and Shadow all artificial sources of illumination are robotic and mobile, but only come out of hiding when needed. Stickon, a 2 m tall newly-arrived visitor to the island encounters a 4 m tall street lamp at night. It approaches to perform its function. Stickon, perceiving a creature with a glowing head coming on, panics, turns, and runs away along the level road they are on. Stickon sprints away at 6 m/s and the street lamp pursues at 12 m/s.

Consider the shadow cast by Stickon onto the road in the light of of the street lamp.



At the instant that the street lamp is 15 m behind Stickon, ...

1. ... how is the length of the shadow changing? *[4]*

SOLUTION. Suppose we call the distance between the street lamp and Stickon x and the length of the shadow y, as in he annotated diagram above. We want to find out $\frac{dy}{dt}$.

We can see that the triangle with sides formed by the street lamp, the road from the street lamp to the tip of the shadow, and the light beam from the street lamp to the tip of the shadow, is a right triangle with height 4 and base $x + y$. The triangle with sides formed by Stickon, the road from Stickon to the tip of the shadow, and the light from the top of Stickon's head to the tip of the shadow, is a right triangle with height 2 and base y. These two triangles are similar since each has a right angle and they share the angle at the tip of the shadow. It follows that

$$\frac{x+y}{4} = \frac{y}{2} \implies \frac{x}{4} + \frac{y}{4} = \frac{y}{2} \implies \frac{x}{4} = \frac{y}{2} - \frac{y}{4} = \frac{y}{4} \implies x = y$$

at every instant.

Note that $\frac{dx}{dt}$ is the rate at which distance between the street lamp and Stickon is changing, so

$$\frac{dx}{dt} = \text{Stickon's speed} - \text{street lamp's speed} = 6 - 12 = -6\ m/s.$$

Since $y = x$ all the time, it follows that the length of the shadow is changing at a rate of $\frac{dy}{dx} = \frac{dx}{dt} = -6\ m/s$ at every instant, including the one where the street lamp is 15 m behind Stickon, *i.e.* when $x = 15$. □

2. ... how quickly is the tip of the shadow moving? *[1]*

SOLUTION. If you think about it for a bit, the tip of the shadow cast by Stickon by the light of the street lamp is the sum of how Stickon is moving and how the length of the shadow is changing. It follows that at every instant the tip of the shadow is moving at a rate of $6 + (-6) = 0\ m/s$ along the road. That is, the tip of the shadow is *not* moving! ■