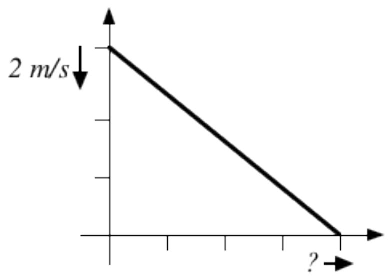
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2025 (S62)

Solution to Quiz #6 – Related Rates
Due on Thursday, 10 July.

1. A smooth horizontal floor meets a smooth vertical wall at a right angle, and a ladder 5 m long is set with its base on the floor and its top against the wall and begins to slide down. At the instant that the top of the ladder is 3 m above the floor, the top is moving down at 2 m/s. How is the distance between the base of the ladder and the wall changing at this instant? *[5]*

SOLUTION. We introduce coordinates so that the x-axis lies along the floor and the y-axis along the wall, as in the diagram below.



The ladder, which is 5 m long, forms the hypotenuse of a right-angled triangle in which the other two sides are (parts of) the floor and the wall. If the top of the ladder is at y and the bottom of the ladder is at x, then $x^2 + y^2 = 5^2 = 25$ by the Pythagorean Theorem. Since $y = 3$ at the instant in question, $x = \sqrt{25 - y^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ at this instant. We are given that $\frac{dy}{dt} = -2$ at the same instant.

To obtain $\frac{dx}{dt}$ at the instant in question, we differentiate both sides of $x^2 + y^2 = 25$,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = \frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}25 = 0\,,$$

plug in what we know of x, y, and $\frac{dy}{dt}$ at the given instant,

$$8\frac{dx}{dt} - 12 = 2 \cdot 4 \cdot \frac{dx}{dt} + 2 \cdot 3 \cdot (-2) = 0\,,$$

and solve for $\frac{dx}{dt}$:

$$8\frac{dx}{dt} - 12 = 0 \quad \Longrightarrow \quad \frac{dx}{dt} = \frac{12}{8} = \frac{3}{2}$$

Thus the ladder is moving away from the wall (as $\frac{dx}{dt} > 0$) at a rate of $\frac{3}{2}$ m/s at the instant in question. ■