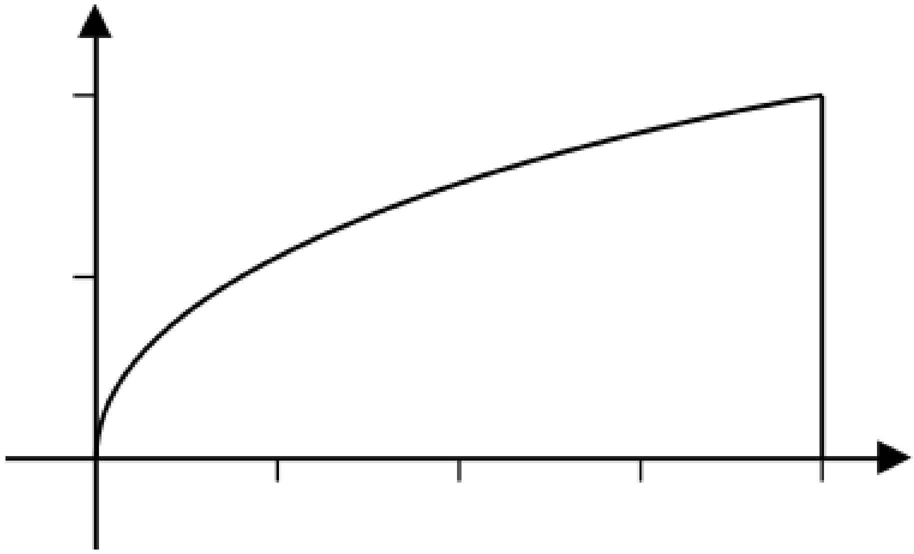
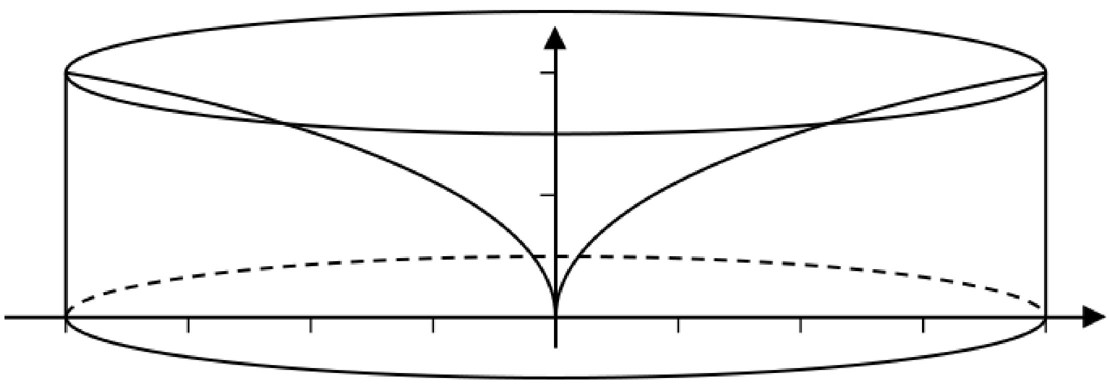
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2025 (S62)
Solutions to Quiz #10 – A Solid of Revolution
*Due on Thursday, 24 July.**

Consider the region between $y = \sqrt{x}$ and $y = 0$, where $0 \le x \le 4$.



1. Sketch the solid obtained by revolving the given region about the y-axis. *[1]*

SOLUTION. Here's a crude sketch of the solid in question:



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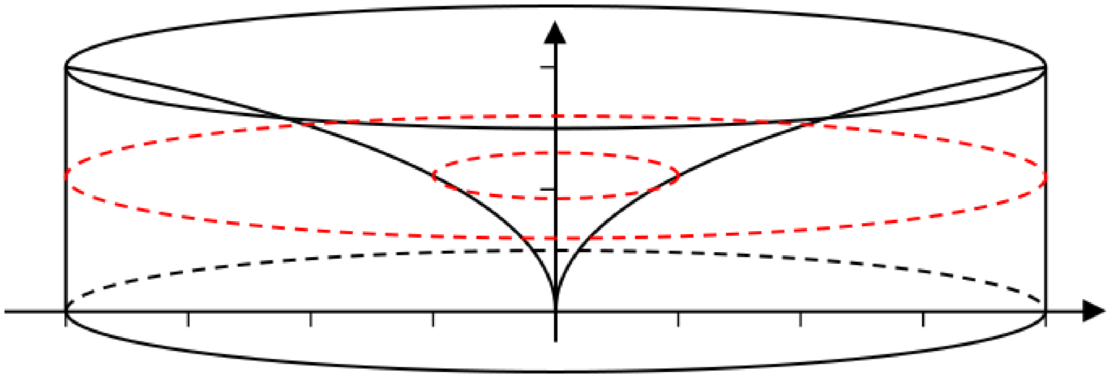
* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper. You may work together and look things up, so long as you write up your submission by yourself and give due credit to your collaborators and any sources you actually used.

2. Find the volume of this solid. *[4]*

SOLUTION 1. *Disk/washer method.* Since we revolved the region about a vertical line and are using disk/washer cross-sections, we should probably use y as the fundamental variable because it is the variable that runs perpendicular to the cross-sections we are using. Note that in terms of y, the given region is the one between $x = 4$ and $x = y^2$, where $0 \leq y \leq 2$.

Here's the sketch of the solid with a washer drawn in:



The outer radius of the washer at y is given by $R = 4$ and the inner radius is given by $r = x = y^2$, so it has area

$$A(y) = \pi R^2 - \pi r^2 = \pi 4^2 - \pi x^2 = 16\pi - \pi \left(y^2\right)^2 = \pi \left(16 - y^4\right).$$

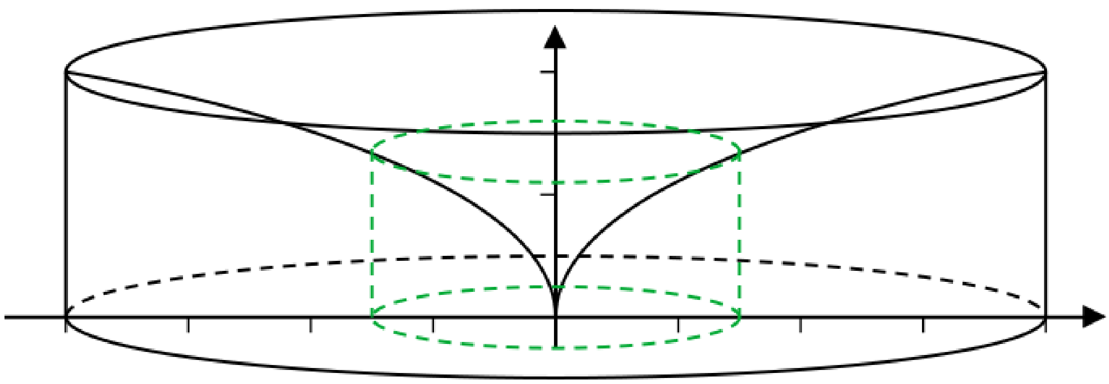
It follows that the volume of the solid is given by

$$\begin{aligned} V &= \int_0^2 A(y)\, dy = \int_0^y \pi \left(16 - y^4\right)\, dy = \pi \left(16y - \frac{y^5}{5}\right)\Bigg|_0^2 \\ &= \pi \left(16 \cdot 2 - \frac{2^5}{5}\right) - \pi \left(16 \cdot 0 - \frac{0^5}{5}\right) = \pi \left(32 - \frac{32}{5}\right) - 0 \\ &= \pi \cdot 32 \cdot \frac{4}{5} = \frac{128\pi}{5}, \end{aligned}$$

in the cube of whatever units are being used for length. □

SOLUTION 2. *Cylindrical shell method.* Since we revolved the region about a vertical line and are using cylindrical shell cross-sections, we should probably use the variable whose axis is perpendicular to these cross-sections, namely x. Recall that in terms of x, the given region is the one between $y = \sqrt{x}$ and $y = 0$, where $0 \leq x \leq 4$.

Here's the sketch of the solid with a cylindrical shell drawn in:



The radius of the cylindrical shell at x is $r = x$ and its height is $h = \sqrt{x} - 0 = x^{1/2}$, so it has area

$$A(x) = 2\pi rh = 2\pi x x^{1/2} = 2\pi x^{3/2}.$$

It follows that the volume of the solid is given by

$$\begin{aligned} V &= \int_0^4 A(x)\,dx = \int_0^4 2\pi x^{3/2}\,dx = 2\pi \cdot \frac{x^{5/2}}{5/2}\bigg|_0^4 = \frac{4\pi}{5} \cdot x^{5/2}\bigg|_0^4 \\ &= \frac{4\pi}{5} \cdot 4^{5/2} - \frac{4\pi}{5} \cdot 0^{5/2} = \frac{4\pi}{5} \cdot 2^5 - 0 = \frac{4\pi}{5} \cdot 32 = \frac{128\pi}{5}, \end{aligned}$$

in the cube of whatever units are being used for length. ■