Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2025 (S62)

Final Examination
19:00-22:00 in ENW 117 on Tuesday, 29 July.

Instructions: Do both of parts **I** and **II**, and, if you wish, part **III**. Please show all your work, justify all your answers, and simplify these where you reasonably can. When you are asked to do k of n questions, only the first k that are not crossed out will be marked. *If you have a question, or are in doubt about something,* **ask!**

Aids: Any calculator, as long as it can't communicate with other devices; (all sides of) one letter- or A4-size sheet; one organic brain belonging to you.

Part I. Do all four (4) of **1**–**4**.

1. Compute $\dfrac{dy}{dx}$ as best you can in any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = \dfrac{9 - x^2}{3 + x}$ **b.** $y = \dfrac{\cos(x)}{1 + \sin(x)}$ **c.** $y = \dfrac{x}{\ln(x)}$

d. $y = (e^x + 3)^5$ **e.** $y = x \tan\left(x^2\right)$ **f.** $y = x^2 e^x$

2. Evaluate any four (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int \frac{x+1}{x^2+1}\,dx$ **b.** $\displaystyle\int_1^e \ln(x)\,dx$ **c.** $\displaystyle\int 6x^2 \cos\left(x^3 + \pi\right)\,dx$

d. $\displaystyle\int_0^1 x^2 e^x\,dx$ **e.** $\displaystyle\int \frac{x+3}{x^2-9}\,dx$ **f.** $\displaystyle\int_0^\pi \sin(2x)\,dx$

3. Do any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Compute $\displaystyle\lim_{x\to\infty} \frac{\ln(x)}{x}$.

b. Use the ε–δ definition of limits to verify that $\displaystyle\lim_{x\to -1} (2x + 3) = 1$.

c. At what point (x, y) does the graph of $y = x^2$ have a tangent line with slope 4?

d. Sketch the region between $y = \cos(x)$ and $y = -\cos(x)$, for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$, and find its area.

e. Let $f(x) = |x|$. Determine whether $f'(x)$ is defined at $x = 0$.

f. Suppose $f'(x) = \cos(x)$ and $f(0) = 2$. What is the function $f(x)$?

4. Find the domain, intercepts, vertical and horizontal asymptotes, intervals of increase and decrease, maximum and minimum points, intervals of concavity, and inflection points of $f(x) = xe^x$. *[15]*

Parts II and III are on page 2.

Part I is on page 1.

Part II. Do one (1) of **5**–**7**.

5. The region between $y = 4$ and $y = 4 - x$, where $0 \leq x \leq 4$, is revolved about the y-axis.

a. Sketch the resulting solid of revolution. *[2]*

b. Find the volume of the solid. *[8]*

6. It is night. Meredith Stick, who is 1.5 m tall, walks slowly at 1 m/s on level ground, holding a lamp on a stick 2 m above the ground. Meredith is moving straight towards a 1 m tall fence post, which casts a shadow on the ground in the light from the lamp.

a. Draw a diagram of this setup. *[2]*

b. How is the length of this shadow changing at the instant that Meredith is 4 m from the post? *[8]*

7. A rectangle has its base on the part of the x-axis with $-4 \leq x \leq 4$, and its upper corners on the lines $y = 4 + x$ and $y = 4 - x$, respectively.

a. Draw a diagram of this setup. *[2]*

b. What is the maximum possible area of such a rectangle? *[8]*

[Total = 85]

Part III. *Here be bonus points!* Do none, or one, or both of the following questions.

$\sqrt{\mathbf{64}}$**.** Suppose you know that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (Which is true.) What does $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$ then have to be? *[1]*

$\sqrt{\mathbf{81}}$**.** Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

Rest, relax, and enjoy the rest of the summer!