Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2025 (S62)

Assignment #3 – A Very Few Differential Equations
*Due on Friday, 11 July.**

Since the midterm test will be written while this assignment is live, it is not intended to be all that challenging, though it does involve learning how to take derivatives and solve equations involving them. If you haven't already, please read Subsections 4.22.1 and 4.22.2 of *Sage for Undergraduates*, before tackling this assignment.

A *first-order linear differential equation* is one of the form $a(x) \cdot \dfrac{dy}{dx} + b(x) \cdot y = c(x)$, which is to be solved for the unknown function(s) y of x that satisfy the equation. It is a differential equation because it involves the derivative of the unknown function; it is first-order because it only involves the first derivative of the unknown function; it is linear because both the unknown function and its derivative occur by themselves in a sum with coefficients that are at most some given functions of x. The equation is further said to be *homogeneous* if the right-hand side function, $c(x)$, is just 0. A requirement that y have a specified value when x has some specified value is said to be an *initial condition* and generally picks out a particular solution from the multiple possible solutions to a given differential eqution.

Differential equations occur in a lot of applications and so have been, and continue to be, extensively studied. Sadly, a lot of them are hard to solve (and some are impossible to solve in nice terms), but linear differential equations are among the happy exceptions. In this assignment you will develop a formula for the solutions of a (narrow!) class of homogeneous first-order linear differential equations with a particular initial condition with the help of SageMath.

1. Use SageMath to solve the homogeneous first-order linear differential equation $\dfrac{dy}{dx} + x^n y = 0$ with initial condition $(x, y) = (0, 1)$ for:
a. $n = 0$ **b.** $n = 1$ **c.** $n = 2$ **d.** $n = 3$ **e.** $n = 4$ *[5 = 5×1 each]*

2. Use the answers you obtained in solving question **1** to guess a solution to $\dfrac{dy}{dx} + x^n y = 0$ with initial condition $(x, y) = (0, 1)$ in terms of x and the integer $n \geq 0$. Verify, by hand, that your solution to thie equation works. *[3]*

3. What happens if you set $n = -1$? Is there a solution for the initial condition $(x, y) = (0, 1)$? Are there any other initial conditions for which you get a solution? *[2]*

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* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper. You may work together and look things up, so long as you write up your submission by yourself and give due credit to your collaborators and any sources you actually used.