Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

Trent University, Summer 2025 (S62)

Assignment #2 – Implicitly Defined Curves

Due on Friday, 4 July.[*]

Since this assignment will go live near the beginning of the extra-long Canada Day weekend, it is not intended to be all that challenging or introduce anything really new. If you haven't already, please read Section 3.4 of *Sage for Undergraduates*, which is about plotting curves implicitly defined by equations in x and y, before tackling this assignment.

Here are five well-studied curves that are defined implicitly. Each equation includes a fixed *parameter* a and perhaps also b; changing the value of a parameter modifies the curve in some way.

Cardioid: $\left(x^2+y^2\right)^2 + 4ax\left(x^2+y^2\right) - 4a^2y^2 = 0$ [For $a > 0$ only.]

Nephroid: $\left(x^2+y^2-4a^2\right)^3 = 108a^4y^2$

Deltoid: $\left(x^2+y^2\right)^2 + 18a^2\left(x^2+y^2\right) - 27a^4 = 8a\left(x^3-3xy^2\right)$

Lemniscate of Bernoulli: $\left(x^2+y^2\right)^2 = a^2\left(x^2-y^2\right)$ [Jakob Bernoulli, that is.]

Limaçon: $\left(x^2+y^2-ax\right)^2 = b^2\left(x^2+y^2\right)$

1. Use SageMath to plot each of the following curves. You will probably need to experiment a bit with the ranges of x and y in each case to ensure that your plot displays the whole curve.

- **a.** The cardioid with parameter $a = 1$. *[1]*
- **b.** The nephroid with parameter $a = 1$. *[1]*
- **c.** The deltoid with parameter $a = 1$. *[1]*
- **d.** The lemniscate of Bernoulli with parameter $a = 1$. *[1]*
- **e.** The limaçon with parameters $a = 1$ and $b = \frac{1}{2}$. *[1]*
- **f.** The limaçon with parameters $a = 1$ and $b = 1$. *[1]*
- **g.** The limaçon with parameters $a = 1$ and $b = 2$. *[1]*

2. Consider the plot of the nephroid you (hopefully!) obtained in part **b** of question **1**.

- **a.** If you decompose the graph of the nephroid into pieces that are each a graph of some continuous function $y = f(x)$ (with limited domain in each case), how many pieces do you need? Indicate what these pieces are with a suitable sketch or sketches. *[2]*
- **b.** If you decompose the graph of the nephroid into pieces that are each a graph of some continuous function $x = g(y)$ (with limited domain in each case), how many pieces do you need? Indicate what these pieces are with a suitable sketch or sketches. *[1]*

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[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper. You may work together and look things up, so long as you write up your submission by yourself and give due credit to your collaborators and any sources you actually used.