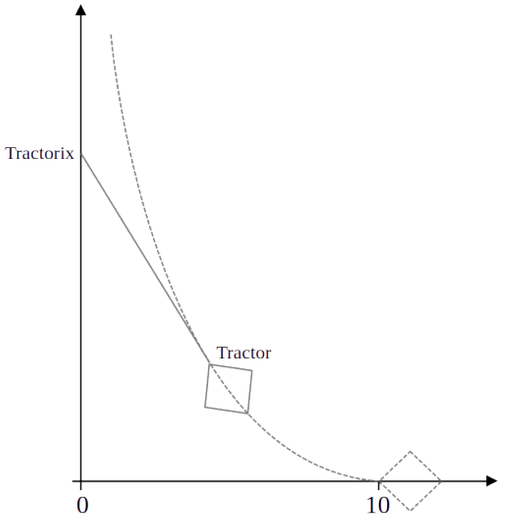
**Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals**
TRENT UNIVERSITY, Summer 2023 (S61)

**Assignment #3**
**Tractor Pull**
*Due* just before midnight on Friday, 26 May.*

Before tackling this assignment, take a peek at the file `1110H-lab-20230517.pdf`, which you can find in the Labs folder in the Course Content section on Blackboard. Skimming and later referring to as necessary to Section 4.22 of Gregory Bard's book *Sage for Undergraduates* (in the SageMath folder in the Course Content section on Blackboard) is probably a good idea. *(Be warned that his book uses a format for declaring unknown functions that is not accepted by recent versions of SageMath. See the lab file above for how to do it right.)* If you wish to use another general purpose mathematics application, such as Maple or Mathematica, you may, but you're on your own for learning to use it and getting help.

In the beginning, Tractorix stands on the edge of a parking lot, holding onto one end of a 10 $m$ cable whose other end is attached to a tractor. At this point the cable is stretched out at right angles to the edge of the parking lot. Tractorix begins to walk along the edge of the parking lot while holding onto the cable and towing the tractor. You may assume that the cable remains straight and taut and 10 $m$ long, and that the path followed by the tractor has the property that the cable is always tangent to this path. (This is an example of a type of curve called a *tractrix*.) See the sketch below to help visualize all this.




Your task will be to determine just what the path taken by the boat is. To help do this, we'll introduce Cartesian coordinates as suggested by the diagram. Let the $y$-axis run along the edge of the parking lot, with the origin at Tractorix's starting location, and with the direction Tractorix walks in being the positive direction, while the rope is initially

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If this fails, you may submit your work to the instructor on paper or by email to `sbilaniuk@ trentu.ca` .

stretched out along the positive $x$-axis. The path followed by the boat will be the graph of $y = f(x)$; note that $f(10) = 0$ and $f'(10) = 0$.

We will find the function $f(x)$ in two steps:

**1.** Find an expression for $\dfrac{dy}{dx} = f'(x)$ in terms of $x$. *[5]*

*Hint:* When the boat is at $(x, f(x))$, the rope is still 10 $m$ long and its slope is $f'(x) = \dfrac{dy}{dx}$.

**2.** Use SageMath to solve the differential equation you obtained in solving **1** for $y = f(x)$, where $f(10) = 0$ and $f'(10) = 0$. *[5]*