Mathematics 1120H – Calculus II: Integrals and Series
Trent University, Summer 2021 (S62)

Assignment #2
The Beta Function
Due on Friday, 2 July.

Submission: Scanned or photographed solutions are fine, so long as they are legible. Please try to make sure that they are oriented correctly – if they are sideways or upside down, they're rather harder to mark online. Submission as a single pdf is strongly preferred, but other common formats are probably OK in a pinch. Please submit your solutions via Blackboard's Assignments module. If Blackboard does not acknowledge a successful upload, please try again. As a *last* resort, email your solutions to the instructor at: `sbilaniuk@trentu.ca`

One of the ways in which integration is used in mathematics is to define functions. One example of this is defining the natural logarithm function by $\ln(x) = \int_1^x \frac{1}{t}\,dt$ for $x > 0$, instead of as the inverse function to e^x. We will look at another example in this assignment, defining the Beta function $\mathbf{B}(x, y)$ for $x > 0$ and $y > 0$ by:

$$\mathbf{B}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$$

Note that as far as evaluating this definite integral is concerned, x and y are treated as constants, since x and y are unaffected by t. The Beta function is pretty useful in various parts of mathematics: the Beta distribution in statistics is defined using this function, it arises in the solutions to various systems of differential equations, has connections with the binomial coefficients, and so on. It can be defined in other ways, but the definition given above is the standard one because it is probably the simplest.

Your task in this assignment will be use what you have learned about integration so far, applied to the definition above, to develop some of the basic facts about the Beta function. In what follows, you may assume that $x > 0$ and $y > 0$ are unknown fixed (*i.e.* constant) real numbers.

1. Verify that $\mathbf{B}(x,y) = \mathbf{B}(y,x)$. *[2]*

2. Verify that $\mathbf{B}(1,y) = \frac{1}{y}$. *[2]*

3. Verify that $\mathbf{B}(x,y+1) + \mathbf{B}(x+1,y) = \mathbf{B}x,y)$. *[2]*

4. Verify that $\mathbf{B}(x+1,y) = \frac{x}{y} \cdot \mathbf{B}(x,y+1)$. *[3]*

5. Use the equations in **3** and **4** to verify that $\mathbf{B}(x+1,y) = \frac{x+y}{x} \cdot \mathbf{B}(x,y)$. *[1]*

[Total = 10]