Mathematics 1120H – Calculus II: Integrals and Series
Trent University, Summer 2020

Solutions to Quiz #2

1. Compute $\displaystyle\int [\ln(x)]^2 \, dx$. Show all your work. *[2.5]*

Solution. This is best done with integration by parts:

$$\begin{aligned}
\int [\ln(x)]^2 \, dx &= [\ln(x)]^2 \cdot x - \int 2\ln(x) \cdot \frac{1}{x} \cdot x \, dx && \begin{matrix} u = [\ln(x)]^2 & v' = 1 \\ u' = 2\ln(x) \cdot \frac{1}{x} & v = x \end{matrix} \\
&= x\,[\ln(x)]^2 - 2\int \ln(x)\, dx && \begin{matrix} s = \ln(x) & t' = 1 \\ s' = \frac{1}{x} & t = x \end{matrix} \\
&= x\,[\ln(x)]^2 - 2\left(\ln(x) \cdot x - \int \frac{1}{x} \cdot x \, dx\right) \\
&= x\,[\ln(x)]^2 - 2\left(x\ln(x) - \int 1 \, dx\right) \\
&= x\,[\ln(x)]^2 - 2\,(x\ln(x) - x) + C \\
&= x\,[\ln(x)]^2 - 2x\ln(x) + 2x + C \qquad \square
\end{aligned}$$

2. Suppose $f(x)$ is a function that is defined and integrable on the interval $[a, b]$. Verify that $\displaystyle\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$. Show all your work. *[2.5]*

Solution. We'll start from the right-hand side and work toward the left-hand side using substitution:

$$\begin{aligned}
\int_a^b f(a+b-x)\, dx &= \int_b^a f(w)\,(-1)dw && \begin{matrix} w = a+b-x, \text{ so } dw = (-1)dx, \text{ and} \\ \text{hence } dx = (-1)dw \text{ and } \begin{matrix} x & a & b \\ w & b & a \end{matrix} \end{matrix} \\
&= -\int_b^a f(u)\, du = \int_a^b f(u)\, du = \int_a^b f(x)\, dx
\end{aligned}$$

... with the last equality holding because it doesn't matter what you call the internal variable in a definite integral. ■