**Mathematics 1120H – Calculus II: Integrals and Series**
Trent University, Winter 2020
**Solutions to Quiz #1**

**1.** Compute $\int_{-1}^{2} \left(3x^2 - 2x + 1\right)\, dx$ using the basic properties of the definite integral, the Fundamental Theorem of Calculus, and the Power Rule for integration. At each step in which you use one of these, indicate which. *[2]*

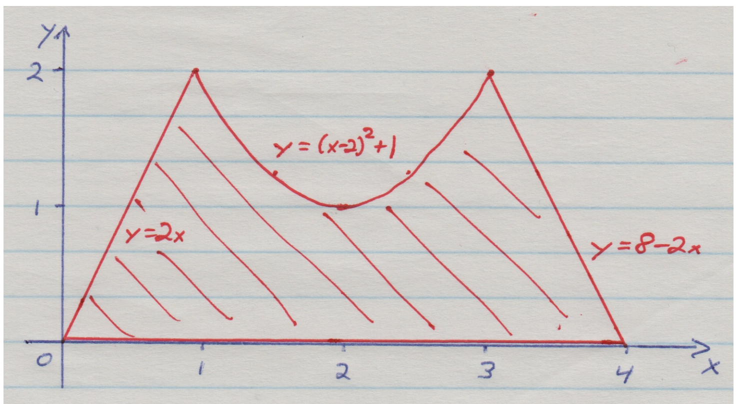
Solution. Here we go:

$$\begin{aligned}
\int_{-1}^{2} \left(3x^2 - 2x + 1\right)\, dx &= \int_{-1}^{2} 3x^2 + \int_{-1}^{2} (-2)x\, dx + \int_{-1}^{2} 1\, dx \\
&\quad \text{(Since integrals are linear, sums pass through.)} \\
&= 3\int_{-1}^{2} x^2\, dx + (-2)\int_{-1}^{2} x^1\, dx + \int_{-1}^{2} x^0\, dx \\
&\quad \text{(Since integrals are linear, multiplication by constants} \\
&\quad \text{ passes through. Also, } x^0 = 1.\text{)} \\
&= 3\frac{x^3}{3}\Big|_{-1}^{2} + (-2)\frac{x^2}{2}\Big|_{-1}^{2} + \frac{x^1}{1}\Big|_{-1}^{2} \\
&\quad \text{(Using the Power Rule and the Fundamental Theorem} \\
&\quad \text{ of Calculus.)} \\
&= x^3\big|_{-1}^{2} - x^2\big|_{-1}^{2} + x\big|_{-1}^{2} \\
&\quad \text{(Just a bit of cancellation to simplify things.)} \\
&= \left[2^3 - (-1)^3\right] - \left[2^2 - (-1)^2\right] + [2 - (-1)] \quad \text{(Evaluation.)} \\
&= [8 - (-1)] - [4 - 1] + 3 = 9 - 3 + 3 = 9 \quad \text{(Arithmetic.)} \quad \square
\end{aligned}$$

Consider the region whose lower boundary is the piece of the $x$-axis for which $0 \le x \le 4$ and whose upper boundary consists of $y = 2x$ for $0 \le x \le 1$, $y = x^2 - 4x + 5$ for $1 \le x \le 3$, and $y = -2x + 8$ for $3 \le x \le 4$.

**2.** Sketch this region. *[1]*

Solution.



Note that $y = x^2 - 4x + 5 = (x-2)^2 + 1$ (via the magic of completing the square), so this curve is a parabola, with its tip at $(2,1)$ and opening upwards. $\square$

**3.** Find the area of this region. *[2]*

SOLUTION. The area of the region is the area between $y = 2x$ and the $x$-axis for $0 \leq x \leq 1$, plus the area between $y = x^2 - 4x + 5$ and the $x$-axis for $1 \leq x \leq 3$, plus the area between $y = -2x + 8$ and the $x$-axis for $3 \leq x \leq 4$. Each of these areas is given by a definite integral; note that none of these sub-regions dip below the $x$-axis, so the corresponding definite integrals compute the actual area in each case.

$$\begin{aligned}
\text{Area} &= \int_0^1 2x\,dx + \int_1^3 \left(x^2 - 4x + 5\right)\,dx + \int_3^4 (8-2x)\,dx \\
&= 2\frac{x^2}{2}\Big|_0^1 + \left(\frac{x^3}{3} - 4\frac{x^2}{2} + 5x\right)\Big|_1^3 + \left(8x - 2\frac{x^2}{2}\right)\Big|_3^4 \\
&= \left[1^2 - 0^2\right] + \left[\left(\frac{3^3}{3} - 2\cdot 3^2 + 5\cdot 3\right) - \left(\frac{1^3}{3} - 2\cdot 1^2 + 5\cdot 1\right)\right] \\
&\quad + \left[\left(8\cdot 4 - 4^2\right) - \left(8\cdot 3 - 3^2\right)\right] \\
&= [1-0] + \left[6 - \frac{10}{3}\right] + [16 - 15] = 1 + \frac{8}{3} + 1 = \frac{14}{3} \qquad \blacksquare
\end{aligned}$$