

**Mathematics 1100Y – Calculus I: Calculus of one variable**

TRENT UNIVERSITY, Summer 2012

**Assignment #11**

**Powerfully series business**

*Due on Wednesday, 1 August, 2012.*

1. Suppose  $x$  is a variable and  $a_n$  for  $n \geq 0$  are constants such that

$$\begin{aligned}\sum_{n=0}^{\infty} a_n x^n &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \\ &= (1 + x + x^2 + x^3 + \dots)^2 = \left( \sum_{n=0}^{\infty} x^n \right)^2.\end{aligned}$$

Find a formula for  $a_n$  in terms of  $n$ . [3]

HINT: Work out the first few  $a_n$ s by multiplying out  $(1 + x + x^2 + x^3 + \dots)^2$  and then collecting like terms, and look for a pattern.

2. Use **Maple** to find the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ . [4]

3. Assume that  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$  no matter what value is given to

the variable  $x$ . There is another power series  $\sum_{n=0}^{\infty} b_n x^n$  such that

$$\left( \sum_{n=0}^{\infty} \frac{x^n}{n!} \right) \left( \sum_{n=0}^{\infty} b_n x^n \right) = 1$$

for every value of the variable  $x$ . Find a formula for  $b_n$  in terms of  $n$ . [3]