Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, SUMMER 2011

Assignment #6
Numerical solutions

Your main tool for **1**, at least in worksheet mode, should be the `int` command, which can be used for both symbolic and numerical integration. The additional tool you will need for **2**, at least in worksheet mode, is the `fsolve` command, which is used to find numerical solutions to equations. Look them up in `Maple`'s help! These commands normally expect you to specify a variable to integrate with respect to or solve for, respectively.

By the way, you can express ∞ in worksheet mode by typing `infinity`.

1. Use `Maple` to compute $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\,dx$. *[5]*

SOLUTION. Worksheet-style:

```
> int(exp(-x^2),x=-infinity..infinity);
```

$$\sqrt{\pi}$$

Surprising, isn't it that this integral would have something to do with π? $\square$

2. Find the value of t such that $\displaystyle\int_{-t}^{t} e^{-x^2}\,dx = \frac{1}{2}$. *[5]*

SOLUTION. Worksheet-style:

```
> fsolve(int(exp(-x^2),x=-t..t)=0.5,t);
```

$$0.2554492865$$

$\square$

Bonus: Recall that a real number is *rational* if it can be written as a ratio of integers, and *irrational* if it is not rational. Show that there are irrational numbers a and b such that a^b is rational. ($a = b$ is allowed, if you can pull it off.) *[1]*

SOLUTION. $\sqrt{2}$ is one of the better-known irrational numbers, so we first try $a = b = \sqrt{2}$. The problem is that it is very hard to tell whether $a^b = \sqrt{2}^{\sqrt{2}}$ is rational or irrational. We can work around this difficulty, though.

If it's rational, we're done.

If it's irrational, however, we can still get the job done another way by setting $a = \sqrt{2}^{\sqrt{2}}$ and $b = \sqrt{2}$. In this case $a^b = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}\cdot\sqrt{2}} = \sqrt{2}^2 = 2$ is a rational number.

Either way, there must be irrational numbers a and b such that a^b is rational. We just don't know which a and b actually do the job ... $\square$