Mathematics 1100Y – Calculus I: Calculus of one variable TRENT UNIVERSITY, SUMMER 2011

Solutions to Assignment #3 Twelve to four

Given twelve identical sticks, each of unit length, it is possible to place them on a flat surface in various ways to form (the perimeters of) polygons (which need not be regular) with integer areas. The figure below shows two such polygons: a square of area nine and a cross with area five.



1. Use all twelve sticks to form (the perimeter of) a polygon of area four. The entire length of each stick must be used, but you can orient them any way you like. [10]

SOLUTION. Consider first the 3-4-5 right triangle formed by using all twelve sticks:



This has area $\frac{1}{2} \cdot 3 \cdot 4 = 6$, which is 2 too much, but it is very easy to reduce the area by 2 by rearranging some stick ts to leave out part of the corner:



Another way to get the job done is to arrange the twelve sticks into a suitable sixpointed star:



The slightly tricky part is to find a convincing argument that there is such a star of area 4 – I'll leave that to you, gentle reader!

There are many other ways to get the job done, too. Look up Martin Gardner's *Nine more problems*, which is Chapter 12 of the first collection of his *Scientific American* columns, *Hexaflexagons and other mathematical diversions*, for some more ways. \Box