# Mathematics 1100Y - Calculus I: Calculus of one variable <br> Trent University, Summer 2011 <br> <br> Solutions to Assignment \#3 <br> <br> Solutions to Assignment \#3 Twelve to four 

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Given twelve identical sticks, each of unit length, it is possible to place them on a flat surface in various ways to form (the perimeters of) polygons (which need not be regular) with integer areas. The figure below shows two such polygons: a square of area nine and a cross with area five.


1. Use all twelve sticks to form (the perimeter of) a polygon of area four. The entire length of each stick must be used, but you can orient them any way you like. [10]
Solution. Consider first the 3-4-5 right triangle formed by using all twelve sticks:


This has area $\frac{1}{2} \cdot 3 \cdot 4=6$, which is 2 too much, but it is very easy to reduce the area by 2 by rearranging some stick ts to leave out part of the corner:


Another way to get the job done is to arrange the twelve sticks into a suitable sixpointed star:


The slightly tricky part is to find a convincing argument that there is such a star of area 4 - I'll leave that to you, gentle reader!

There are many other ways to get the job done, too. Look up Martin Gardner's Nine more problems, which is Chapter 12 of the first collection of his Scientific American columns, Hexaflexagons and other mathematical diversions, for some more ways.

