Mathematics 1100Y – Calculus I: Calculus of one variable TRENT UNIVERSITY, Summer 2010

Assignment #10

Lissajous curves and a bit of Maple

Due on Wednesday, 28 July, 2010.

Lissajous curves are parametric curves of the form $\begin{cases} x = A\cos(at) \\ y = B\sin(bt) \end{cases}$, where t is the parameter and A, B, a, and b are constants. (This is not the most general definition, but let's keep some things simple.) Various common curves can be realized as Lissajous curves.

For example, if $a = b \neq 0$, then one gets an ellipse; if also A = B, a circle. In what follows we will assume, unless stated otherwise, that A = B = 1, that a and b are both positive integers, and that $0 \leq t \leq 2\pi$.

Lissajous curves describe complex harmonic motions and can arise, for example, in problems in orbital mechanics and in signal processing. They are named after Jules Antoine Lissajous, a French mathematician who studied them extensively in the mid-19th Century, though they had received some attention earlier.

Before attempting question 1 below, please read the handout A very quick introduction to Maple, which is mostly pointers to places to learn about Maple, and play around with Maple a little.

1. Use Maple to plot the Lissajous curves for the combinations of a and b given in the following table:

3 224 3 1 24 a21 23 4 2b 4 3

What can you deduce about the behaviour of Lissajous curves for different choices of a and b from these examples? Feel free to plot Lissajous curves for additional choices of a and b to test and/or support your deductions. [5]

2. Is it possible for a Lissajous curve to describe (part of) a parabola? If so, give an example; if not, explain why not. [5]

Note: If you can make use of Maple for 2, more power to you, but you don't need to.