Name: _Solutions_ **Student Number:** ______________

Mathematics 1110H, Trent University, Fall 2025.

Test #5 for Sections F03 and F05. *Friday, 28 November.* Time: 30 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do any four (4) of questions 1–6.**

1. $\displaystyle\int_1^2 \frac{x^2-x-2}{x+1}\,dx$ **2.** $\displaystyle\int e^x\cos(x)\,dx$ **3.** $\displaystyle\int_0^{\pi/4} 2\tan(x)\sec^2(x)\,dx$

4. $\displaystyle\int_0^{\pi/2} \cos^3(x)\,dx$ **5.** $\displaystyle\int x\ln\left(x^2\right)\,dx$ **6.** $\displaystyle\int_0^2 \frac{4x}{\sqrt{4+x^2}}\,dx$

Solutions. **1.** We will factor the numerator to help simplify the integrand, and then use the power rule for integration.

$$\begin{aligned}\int_1^2 \frac{x^2-x-2}{x+1}\,dx &= \int_1^2 \frac{(x+1)(x-2)}{x+1}\,dx = \int_1^2 (x-2)\,dx \\ &= \left(\frac{x^2}{2}-2x\right)\Big|_1^2 = \left(\frac{2^2}{2}-2\cdot 2\right)-\left(\frac{1^2}{2}-2\cdot 1\right) \\ &= (2-4)-\left(\frac{1}{2}-2\right) = -2-\left(-\frac{3}{2}\right) = -\frac{1}{2} \quad \square\end{aligned}$$

2. We will use integration by parts twice and do a little algebra afterwards.

$$\int e^x\cos(x)\,dx = e^x\sin(x) - \int e^x\sin(x)\,dx$$

Parts with $u=e^x$ and $v'=\cos(x)$, so $u'=e^x$ and $v=\sin(x)$.

$$= e^x\sin(x) - \left[e^x\left(-\cos(x)\right) - \int e^x\left(-\cos(x)\right)\,dx\right]$$

Parts with $s=e^x$ and $t'=\sin(x)$, so $s'=e^x$ and $t=-\cos(x)$.

$$= e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)\,dx$$

Moving the $\int e^x\cos(x)\,dx$ at the end to the beginning, we get that $2\int e^x\cos(x)\,dx = e^x\sin(x)+e^x\cos(x)$. It follows that

$$\int e^x\cos(x)\,dx = \frac{e^x\sin(x)+e^x\cos(x)}{2}+C. \quad \square$$

3. We will use the substitution $w=\sec(x)$, so $dw=\sec(x)\tan(x)\,dx$, and change the limits as we go: $\begin{smallmatrix} x & 0 & \pi/4 \\ w & 1 & \sqrt{2}\end{smallmatrix}$ (Since $\cos(\pi/4)=1/\sqrt{2}$, $\sec(\pi/4)=1/\cos(\pi/4)=1/\left(1/\sqrt{2}\right)=\sqrt{2}$.)

$$\begin{aligned}\int_0^{\pi/4} 2\tan(x)\sec^2(x)\,dx &= 2\int_0^{\pi/4}\sec(x)\sec(x)\tan(x)\,dx = \int_1^{\sqrt{2}} w\,dw = \frac{w^2}{2}\Big|_1^{\sqrt{2}} \\ &= \frac{\left(\sqrt{2}\right)^2}{2}-\frac{1^2}{2} = \frac{2}{2}-\frac{1}{2} = \frac{1}{2} \quad \square\end{aligned}$$

4. We will use the identity $\cos^2(x) = 1 - \sin^2(x)$ and the substitution $w = \sin(x)$, so $dw = \cos(x)\,dx$, and change the limits as we go: $\begin{smallmatrix} x & 0 & \pi/2 \\ w & 0 & 1 \end{smallmatrix}$

$$\begin{aligned}
\int_0^{\pi/2} \cos^3(x)\,dx &= \int_0^{\pi/2} \cos^2(x)\cos(x)\,dx = \int_0^{\pi/2} \left(1 - \sin^2(x)\right)\cos(x)\,dx \\
&= \int_0^1 \left(1 - w^2\right)\,dw = \left(w - \frac{w^3}{3}\right)\Big|_0^1 = \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) \\
&= \frac{2}{3} - 0 = \frac{2}{3} \quad \square
\end{aligned}$$

5. We will first simplify the integrand a little, and then use integration by parts with $u = \ln(x)$ and $v' = x$, so $u' = \frac{1}{x}$ and $v = \frac{x^2}{2}$.

$$\begin{aligned}
\int x \ln\left(x^2\right)\,dx &= \int x \cdot 2\ln()\,dx = 2\int x\ln(x)\,dx = 2\left[\frac{x^2}{2}\ln(x) - \int \frac{1}{x} \cdot \frac{x^2}{2}\,dx\right] \\
&= x^2\ln(x) - \int x\,dx = x^2\ln(x) - \frac{x^2}{2} + C \quad \square
\end{aligned}$$

6. We will use the substitution $w = 4 + x^2$, so $dw = 2x\,dx$, and change the limits as we go: $\begin{smallmatrix} x & 0 & 2 \\ w & 4 & 8 \end{smallmatrix}$

$$\begin{aligned}
\int_0^2 \frac{4x}{\sqrt{4+x^2}}\,dx &= \int_4^8 \frac{2}{\sqrt{w}}\,dw = \int_4^8 2w^{-1/2}\,dw = 2 \cdot \frac{w^{1/2}}{1/2}\Big|_4^8 = 4\sqrt{w}\Big|_4^8 \\
&= 4\sqrt{8} - 4\sqrt{4} = 4 \cdot 2\sqrt{2} - 4 \cdot 2 = 8\sqrt{2} - 8 = 8\left(\sqrt{2} - 1\right) \quad \blacksquare
\end{aligned}$$