Name: *Solutions* **Student Number:** ______________

Mathematics 1110H, Trent University, Fall 2025.

Test #5 for Sections F02 and F04. *Friday, 28 November.* Time: 30 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do any four (4) of questions 1–6.**

1. $\displaystyle\int_0^3 (x-1)(x+3)\,dx$ **2.** $\displaystyle\int xe^{x-1}\,dx$ **3.** $\displaystyle\int_0^{\pi/8} \tan(2x)\,dx$

4. $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}\,dx$ **5.** $\displaystyle\int \frac{x}{\sqrt{41-x^2}}\,dx$ **6.** $\displaystyle\int_0^1 4xe^{x^2}\,dx$

Solutions. **1.** We will multiply out the factors of the integrand and then use the power rule for integration.

$$\begin{aligned}\int_0^3 (x-1)(x+3)\,dx &= \int_0^3 \left(x^2+2x-3\right)\,dx = \left(\frac{x^3}{3}+x^2-3x\right)\Big|_0^3 \\ &= \left(\frac{3^3}{3}+3^2-3\cdot 3\right)-\left(\frac{0^3}{3}+0^2-3\cdot 0\right) \\ &= (9+9-9)-(0+0-0) = 9-0 = 9 \quad \square\end{aligned}$$

2. We will rewrite $e^{x-1} = \dfrac{e^x}{e}$ and then use integration by parts with $u = x$ and $v' = e^x$, so $u' = 1$ and $v = e^x$.

$$\begin{aligned}\int xe^{x-1}\,dx &= \int x\frac{e^x}{e}\,dx = \frac{1}{e}\int xe^x\,dx = \frac{1}{e}\left[xe^x - \int 1e^x\,dx\right] = \frac{1}{e}\left[xe^x - e^x\right] + C \\ &= \frac{1}{e}xe^x - \frac{1}{e}e^x + C = xe^{x-1} - e^{x-1} + C = (x-1)e^{x-1} + C \quad \square\end{aligned}$$

3. We will first use the substitution $w = 2x$, so $dw = 2\,dx$ and thus $dx = \frac{1}{2}\,dw$, changing the limits as we go along: $\begin{smallmatrix} x & 0 & \pi/8 \\ w & 0 & \pi/4 \end{smallmatrix}$

$$\int_0^{\pi/8} \tan(2x)\,dx = \int_0^{\pi/4} \tan(w)\,\frac{1}{2}\,dw = \frac{1}{2}\int_0^{\pi/4} \frac{\sin(w)}{\cos(w)}\,dw$$

At this point we will use the substitution $z = \cos(w)$, so $dz = -\sin(w)\,dw$ and thus $\sin(w)\,dw = (-1)dz$, changing the limits as we go along: $\begin{smallmatrix} w & 0 & \pi/4 \\ z & 1 & 1/\sqrt{2} \end{smallmatrix}$

$$\begin{aligned}\int_0^{\pi/8} \tan(2x)\,dx &= \frac{1}{2}\int_0^{\pi/4} \frac{\sin(w)}{\cos(w)}\,dw = \frac{1}{2}\int_1^{1/\sqrt{2}} \frac{1}{z}\,(-1)\,dz = \frac{1}{2}\int_{1/\sqrt{2}}^1 \frac{1}{z}\,dz \\ &= \frac{1}{2}\ln(z)\Big|_{1/\sqrt{2}}^1 = \frac{1}{2}\ln(1) - \frac{1}{2}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}\cdot 0 - \frac{1}{2}\ln\left(2^{-1/2}\right) \\ &= 0 - \frac{1}{2}\left(-\frac{1}{2}\right)\ln(2) = \frac{\ln(2)}{4} \quad \square\end{aligned}$$

4. We will use the substitution $w = \sin(x)$, so $dw = \cos(x)\,dx$, and change the limits as we go along: $\begin{smallmatrix} x & 0 & \pi/2 \\ w & 0 & 1 \end{smallmatrix}$

$$\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}\,dx = \int_0^1 \frac{1}{1+w^2}\,dw = \arctan(w)\big|_0^1$$
$$= \arctan(1) - \arctan(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4} \quad \square$$

5. We will use the substitution $w = 41 - x^2$, so $dw = -2x\,dx$ and thus $x\,dx = \left(-\frac{1}{2}\right) dw$.

$$\int \frac{x}{\sqrt{41-x^2}}\,dx = \int \frac{1}{\sqrt{(w)}}\left(-\frac{1}{2}\right) dw = -\frac{1}{2}\int w^{-1/2}\,dw = -\frac{1}{2}\cdot\frac{w^{3/2}}{3/2} + C$$
$$= -\frac{w^{3/2}}{3} + C = -\frac{\left(41-x^2\right)^{3/2}}{3} + C$$

6. We will use the substitution $w = x^2$, so $dw = 2x\,dx$, and "change" the limits as we go along: $\begin{smallmatrix} x & 0 & 1 \\ w & 0 & 1 \end{smallmatrix}$

$$\int_0^1 4xe^{x^2}\,dx = \int_0^1 2e^w\,dw = 2e^1 - 2e^0 = 2e - 2\cdot 1 = 2(e-1) \quad \blacksquare$$