Name: *Solutions* **Student Number:** ____________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025.

Test #5 for Section F01 and CAT. *Friday, 28 November.* Time: 30 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do any four (4) of questions 1–6.**

1. $\displaystyle\int \cos(2x+1)\,dx$ **2.** $\displaystyle\int xe^{-x}\,dx$ **3.** $\displaystyle\int_0^{\pi/4} \cos(2x)\,dx$

4. $\displaystyle\int_e^{e^e} x\ln(x)\,dx$ **5.** $\displaystyle\int x\sqrt{1-x^2}\,dx$ **6.** $\displaystyle\int_0^2 \left(2x+e^x\right)\,dx$

SOLUTIONS. **1.** We will use the substitution $w = 2x+1$, so $dw = 2dx$ and thus $dx = \frac{1}{2}dw$.

$$\int \cos(2x+1)\,dx = \int \cos(w)\,\frac{1}{2}dw = \frac{1}{2}\sin(w) + C = \frac{1}{2}\sin(2x+1) + C \quad \square$$

2. We will use integration by parts with $u = x$ and $v' = e^{-x}$, so $u' = 1$ and $v = -e{-}x$. We get that $v = -e^{-x}$ via the substitution $w = -x$, so $dw = (-1)dx$ and thus $dx = (-1)dw$: $v = \displaystyle\int e^{-x}\,dx = \int e^w\,(-1)dw = -e^w = -e^{-x}$.

$$\int xe^{-x}\,dx = -xe^{-x} - \int 1\,(-e{-}x)\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C \quad \square$$

3. We will use the substitution $w = 2x$, so $dw = 2dx$ and thus $dx = \frac{1}{2}dw$, and change the limits as we go along: $\begin{smallmatrix} x & 0 & \pi/4 \\ w & 0 & \pi/2 \end{smallmatrix}$

$$\begin{aligned}\int_0^{\pi/4} \cos(2x)\,dx &= \int_0^{\pi/2} \cos(w)\,\frac{1}{2}dw = \frac{1}{2}\sin(w)\Big|_0^{\pi/2} \\ &= \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin(0) = \frac{1}{2}\cdot 1 - \frac{1}{2}\cdot 0 = \frac{1}{2} \quad \square\end{aligned}$$

4. We will use integration by parts with $u = \ln(x)$ and $v' = x$, so $u' = \dfrac{1}{x}$ and $v = \dfrac{x^2}{2}$.

$$\begin{aligned}\int_e^{e^e} x\ln(x)\,dx &= \frac{x^2\ln(x)}{2}\Big|_e^{e^e} - \int_e^{e^e} \frac{1}{x}\cdot\frac{x^2}{2}\,dx = \frac{\left(e^e\right)^2\ln\left(e^e\right)}{2} - \frac{e^2\ln\left(e\right)}{2} - \frac{1}{2}\int_e^{e^e} x\,dx \\ &= \frac{e^{2e}e\ln(e)}{2} - \frac{e^2\cdot 1}{2} - \frac{1}{2}\cdot\frac{x^2}{2}\Big|_e^{e^e} = \frac{e^{3e}\cdot 1}{2} - \frac{e^2}{2} - \left[\frac{\left(e^e\right)^2}{4} - \frac{e^2}{4}\right] \\ &= \frac{e^{3e}}{2} - \frac{e^2}{2} - \frac{e^{2e}}{4} + \frac{e^2}{4} = \frac{e^{3e}}{2} - \frac{e^{2e}}{4} - \frac{e^2}{4} \quad \square\end{aligned}$$

5. We will use the substitution $w = 1 - x^2$, so $dw = -2x\,dx$ and thus $x\,dx = \left(-\frac{1}{2}\right)\,dw$.

$$\begin{aligned}\int x\sqrt{1-x^2}\,dx &= \int \sqrt{w}\left(-\frac{1}{2}\right)\,dw = -\frac{1}{2}\int w^{1/2}\,dw = -\frac{1}{2}\cdot\frac{w^{3/2}}{3/2} + C \\ &= -\frac{w^{3/2}}{3} + C = -\frac{\left(1-x^2\right)^{3/2}}{3} + C \quad \square\end{aligned}$$

6. We will use the sum and power rules for integration.

$$\begin{aligned}\int_0^2 \left(2x + e^x\right)\,dx &= \left(2\cdot\frac{x^2}{2} + e^x\right)\Bigg|_0^2 = \left(x^2 + e^x\right)\Big|_0^2 \\ &= \left(2^2 + e^2\right) - \left(0^2 + e^0\right) = 4 + e^2 - 1 = e^2 + 3 \quad \blacksquare\end{aligned}$$