Name: _Solutions_ **Student Number:** ____________

Mathematics 1110H, Trent University, Fall 2025.

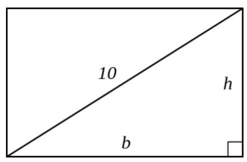
Test #4 for Sections F02 and F04. *Friday, 14 November.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do one (1) of questions 1 or 2.** (Question **2** is on page 2.)

1. What is the maximum area of a rectangle whose diagonal is 10 *cm* long? *[10]*

Solution. A rectangle with base b, height h, and diagonal 10 has area $A = bh$ and, because the diagonal and two of the sides of the rectangle form a right triangle, satisfies $b^2 + h^2 = 10^2 = 100$ by the Pythagorean Theorem.



We can use this relationship to solve for b in terms of h:

$$b^2 + h^2 = 100 \implies b^2 = 100 - h^2 \implies b = \sqrt{100 - h^2}$$

(We take the positive square root because b is a length.) It follows that the area is, in terms of h: $A(h) = bh = h\sqrt{100 - h^2}$. Note that we must have $0 \leq h \leq 10$ for the area to be defined and positive. At the endpoints we have area $A(0) = 0\sqrt{100 - 0^2} = 0 \cdot 10 = 0$ and $A(10) = 10\sqrt{100 - 10^2}10 \cdot 0 = 0$.

We next check to see if there is a critical point in the interval $[0, 10]$:

$$\begin{aligned}
A'(h) &= \frac{d}{dh}\left(h\sqrt{100 - h^2}\right) = \left[\frac{dh}{dh}\right]\sqrt{100 - h^2} + h\left[\frac{d}{dh}\sqrt{100 - h^2}\right] \\
&= 1\sqrt{100 - h^2} + h \cdot \frac{1}{2\sqrt{100 - h^2}} \cdot \frac{d}{dh}\left(100 - h^2\right) \\
&= \sqrt{100 - h^2} + \frac{h}{2\sqrt{100 - h^2}} \cdot (-2h) = \sqrt{100 - h^2} - \frac{h^2}{\sqrt{100 - h^2}} = 0 \\
&\implies 100 - h^2 - h^2 = 100 - 2h^2 = 0 \implies h^2 = 50 \implies h = \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}
\end{aligned}$$

$h = 5\sqrt{2} \approx 7.07106$ is in the interval $[0, 10]$ and

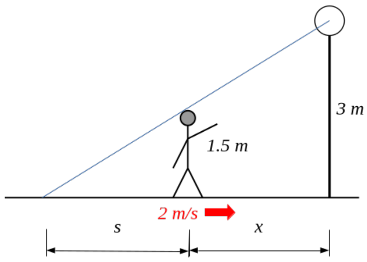
$$A(5) = 5\sqrt{100 - \left(5\sqrt{2}\right)^2} = 5\sqrt{(100 - 50)} = 5\sqrt{50} = 5 \cdot 5\sqrt{2} = 25\sqrt{2} \approx 35.35534\,.$$

Since the area at the critical point is greater than that at the endpoints, it is the maximum.

Thus the maximum area of a rectangle with a diagonal of 10 *cm* is $25\sqrt{2}$ cm^2. ■

2. Sticklike Figure, who is 1.5 m tall, walks at 2 m/s on level ground at night, straight towards a 3 m tall lamppost that is lit up. How fast is the tip of Sticklike's shadow moving along the ground at the instant that Sticklike is 6 m from the lamppost? *[10]*

SOLUTION. Let x be the distance between Sticklike and the lamppost, and s be the length of the shadow cast by Sticklike.



We are given that $\frac{dx}{dt} = -2\ m/2$ – it's negative because Sticklike is walking towards the lamppost – and asked to find $\frac{d}{dt}(x+s)$ – the tip of the shadow's motion is a combination of Sticklike's motion and the changing length of the shadow – at the instant that $x = 6\ m$.

Using the similar triangles evident in the diagramm we have that s is to 1.5 as $s + x$ is to 3, *i.e.*

$$\frac{s}{1.5} = \frac{x+s}{3} \implies 2s = x + s \implies s = x.$$

It follows that $\frac{ds}{dt} = \frac{dx}{dt} = -2$, and thus the tip of the shadow is moving at a rate of

$$\frac{d}{dt}(x+s) = \frac{ds}{dt} + \frac{dx}{dt} = -2 - 2 = -4\ m/s.$$

It's negative because it is approaching the lamppost. If all you want is speed – no direction! – then it is moving at 4 m/s along the ground. ■