Name: *Solutions* **Student Number:** ________________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025.

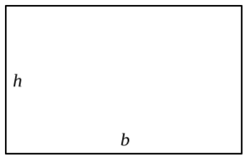
Test #4 for Sections F02 and F04. *Friday, 14 November.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do one (1) of questions 1 or 2.** (Question **2** is on page 2.)

1. What is the maximum area of a rectangle whose perimeter is 12 *cm* long? *[10]*

SOLUTION. A rectangle with base b and height h has area $A = bh$ and perimeter $P = 2b + 2h$.



We are asked to maximize the area A subject to the constraint that the perimeter $P = 2b + 2h = 12$ *cm*. Note that this constraint implies that $0 \leq h \leq 6$ and $0 \leq b \leq 6$. We can also use the constraint to solve for one of b and h in terms of the other, *e.g.*

$$2b + 2h = 12 \implies b + h = 6 \implies h = 6 - b.$$

The given problem thus reduces to maximizing $A(b) = bh = b(6-b) = 6b-b^2$ for $0 \leq b \leq 6$.

First, observe that at the endpoints the area is 0: $A(0) = 0(6-0) = 0$ and $A(6) = 6(6-6) = 6 \cdot 0 = 0$.

Second, we look for critical points in the given interval:

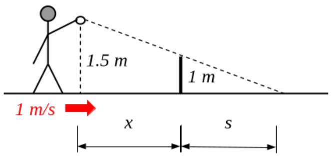
$$A'(b) = \frac{d}{db}\left(6b - b^2\right) = 6 - 2b = 0 \implies 6 = 2b \implies b = \frac{6}{2} = 3$$

$b = 3$ is between 0 and 6, so we check the area at this critical point: $A(3) = 3(6-3) = 3 \cdot 3 = 9$. This is greater than the value at the endpoints, so it is the maximum of $A(b)$ for $0 \leq b \leq 6$.

Thsus the maximum area of a rectangle with perimeter 12 *cm* is 9 cm^2. ■

2. Stick Person takes a walk on a dark night. Stick walks slowly at 1 m/s on level ground, holding a lamp 1.5 m above the ground, straight towards a 1 m tall post, which casts a shadow on the ground in the light from the lamp. How is the length of this shadow changing at the instant that Stick is 3 m from the post? *[10]*

SOLUTION. Here is sketch of the setup, with the horizontal between the lamp and the stick being x and the length of the shadow being s.



We are given that $\frac{dx}{dt} = -1\ m/s$ – it's negative because Stick Person is approaching the post, thus making x smaller with time – and asked to find $\frac{ds}{dt}$ at the instant that $x = 3\ m$.

Using the similar triangles evident in the diagram, we have that s is to 1 as $x + s$ is to 1.5, *i.e.*

$$\frac{s}{1} = \frac{x+s}{1.5} \implies 1.5s = x + s \implies x = 0.5s \implies s = 2x.$$

It follows that

$$\frac{ds}{dt} = \frac{d}{dt}(2x) = 2\frac{dx}{dt} = 2(-1) = -2,$$

which is true at every instant, including when $x = 3$.

Thus the length of the shadow is changing at rate of $-2\ m/s$ at the instant that $x = 3\ m$, and at every other instant, too. ■